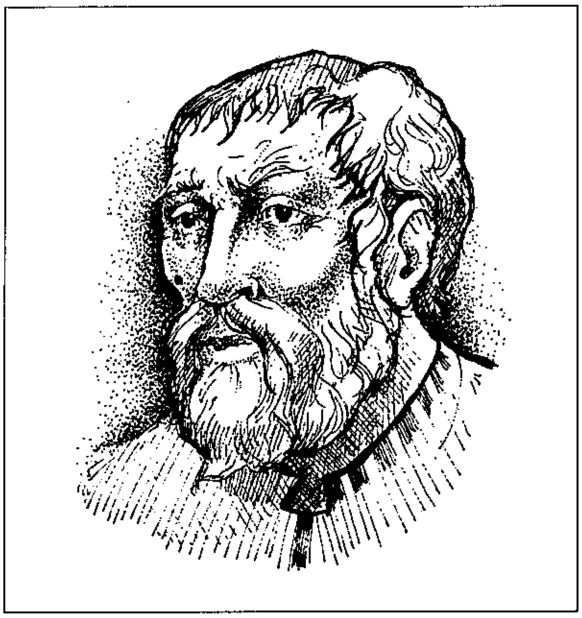
4 GEOMETRY



Pythagoras (569–500 B.C.) was one of the first of the great mathematical names in Greek antiquity. He settled in southern Italy and formed a mysterious brotherhood with his students who were bound by an oath not to reveal the secrets of numbers and who exercised great influence. They laid the foundations of arithmetic through geometry but failed to resolve the concept of irrational numbers. The work of these and others was brought together by Euclid at Alexandria in a book called 'The Elements' which was still studied in English schools as recently as 1900.

26 Use and interpret geometrical terms, including similarity and congruence; use the relationships between areas and volumes of similar figures; use and interpret vocabulary of shapes and simple solid figures including nets

27 Measure lines and angles; construct a triangle given three sides; construct angle bisectors and perpendicular bisectors

28 Recognise rotational and line symmetry

29 Calculate unknown angles using geometrical properties, including irregular polygons and circle theorems

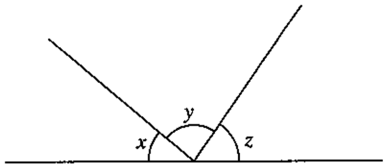
30 Use loci in two dimensions

32 Apply Pythagoras' theorem

4.1 Fundamental results

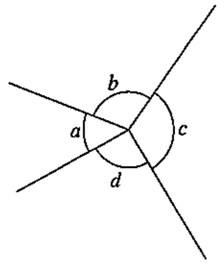
You should already be familiar with the following results. They are used later in this section and are quoted here for reference.

- The angles on a straight line add up to 180°:



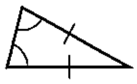
$$\hat{x} + \hat{y} + \hat{z} = 180°$$

- The angles at a point add up to 360°:

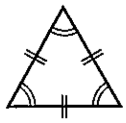


$$\hat{a} + \hat{b} + \hat{c} + \hat{d} = 360°$$

- The angle sum of a triangle is 180°.
- An isosceles triangle has 2 sides and 2 angles the same:



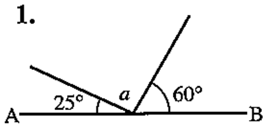
- The angle sum of a quadrilateral is 360°.
- An equilateral triangle has 3 sides and 3 angles the same:



Exercise 1

Find the angles marked with letters. (AB is always a straight line.)

1.



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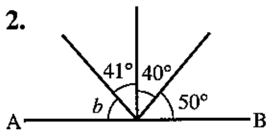

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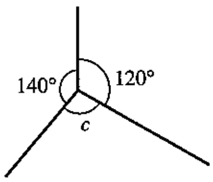

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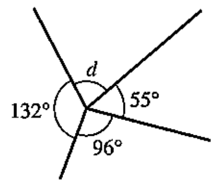

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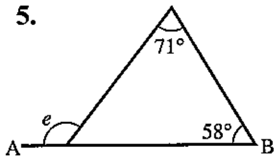

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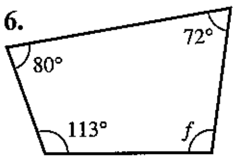

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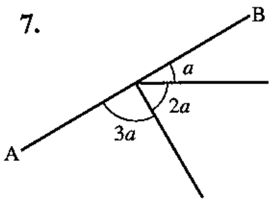

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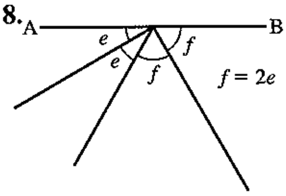

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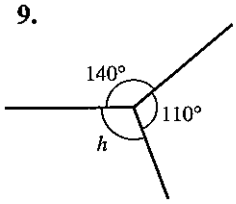

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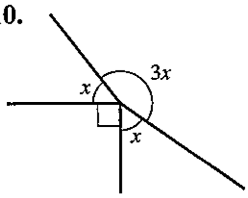

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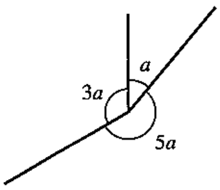

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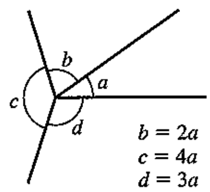

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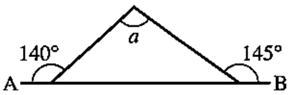

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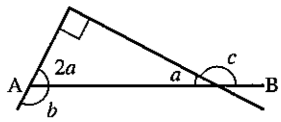

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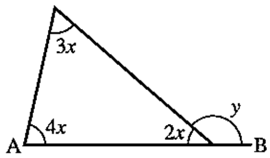

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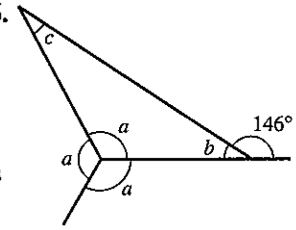

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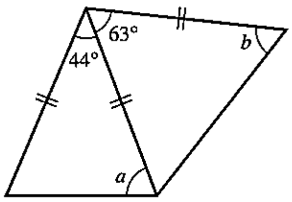

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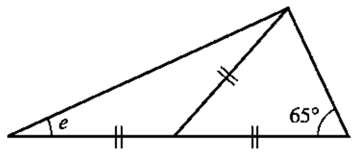

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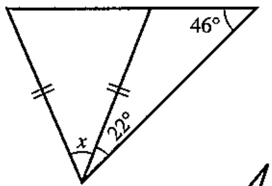

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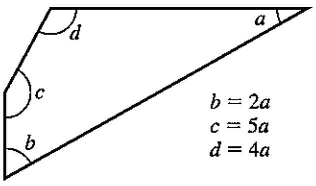

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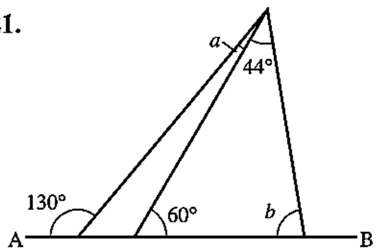

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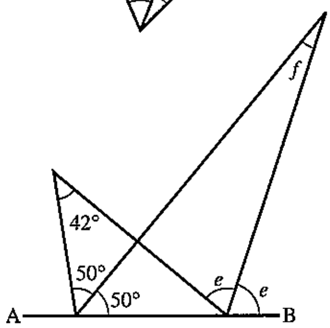


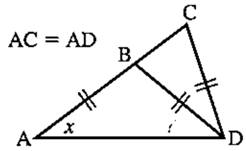
23. Calculate the largest angle of a triangle in which one angle is eight times each of the others.

24. In $\triangle ABC$, $\hat{A}$ is a right angle and D is a point on AC such that BD bisects $\hat{B}$. If $B\hat{D}C = 100°$, calculate $\hat{C}$.

25. WXYZ is a quadrilateral in which $\hat{W} = 108°$, $\hat{X} = 88°$, $\hat{Y} = 57°$ and $W\hat{X}Z = 31°$. Calculate $W\hat{Z}X$ and $X\hat{Z}Y$.

26. In quadrilateral ABCD, AB produced is perpendicular to DC produced. If $\hat{A} = 44°$ and $\hat{C} = 148°$, calculate $\hat{D}$ and $\hat{B}$.

27. Triangles ABD, CBD and ADC are all isosceles. Find the angle x.

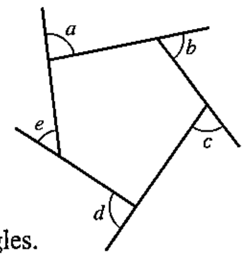


Polygons

(i) The exterior angles of a polygon add up to 360° $(\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} = 360°)$.

(ii) The sum of the interior angles of a polygon is $(n - 2) \times 180°$ where n is the number of sides of the polygon.
This result is investigated in question **3** in the next exercise.

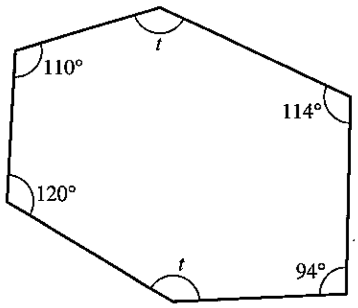
(iii) A regular polygon has equal sides and equal angles.



Remember
pentagon = 5 sides
hexagon = 6 sides
octagon = 8 sides
decagon = 10 sides

Example

Find the angles marked with letters.



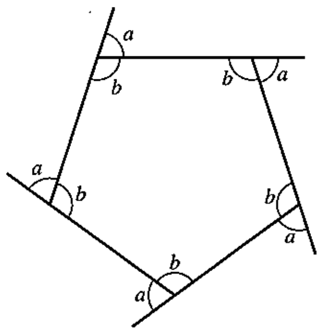
The sum of the interior angles $= (n-2) \times 180°$ where n is the number of sides of the polygon.

In this case $n = 6$.

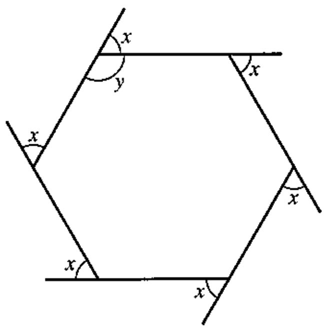
$$
\begin{aligned}
\therefore \quad 110 + 120 + 94 + 114 + 2t &= 4 \times 180 \\
438 + 2t &= 720 \\
2t &= 282 \\
t &= 141°
\end{aligned}
$$

Exercise 2

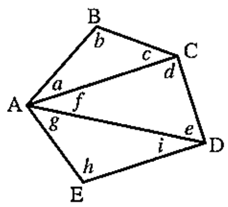
1. Find angles a and b for the regular pentagon.



2. Find x and y.



3. Consider the pentagon below which has been divided into three triangles.



$\widehat{A} = a + f + g$, $\widehat{B} = b$, $\widehat{C} = c + d$, $\widehat{D} = e + i$, $\widehat{E} = h$

Now $a + b + c = d + e + f = g + h + i = 180°$

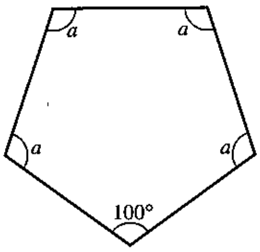
$$
\begin{aligned}
\therefore \quad \widehat{A} + \widehat{B} + \widehat{C} + \widehat{D} + \widehat{E} &= a + b + c + d + e \\
&\quad + f + g + h + i \\
&= 3 \times 180° \\
&= 6 \times 90°
\end{aligned}
$$

Draw further polygons and make a table of results.

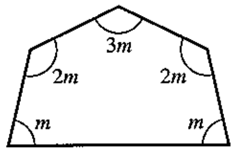
Number of sides n	5	6	7	8...
Sum of interior angles	$3 \times 180°$			

What is the sum of the interior angles for a polygon with n sides?

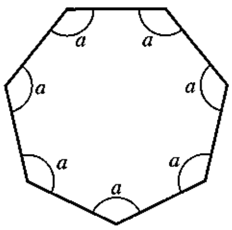
4. Find a.



5. Find m.



6. Find a.



7. Calculate the number of sides of a regular polygon whose interior angles are each 156°.

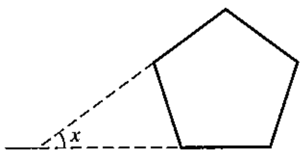
8. Calculate the number of sides of a regular polygon whose interior angles are each 150°.

9. Calculate the number of sides of a regular polygon whose exterior angles are each 40°.

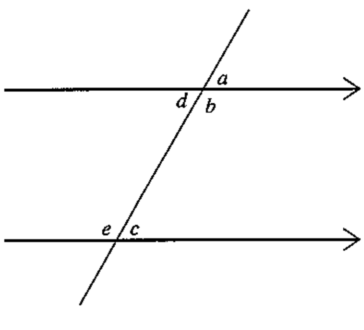
10. In a regular polygon each interior angle is 140° greater than each exterior angle. Calculate the number of sides of the polygon.

11. In a regular polygon each interior angle is 120° greater than each exterior angle. Calculate the number of sides of the polygon.

12. Two sides of a regular pentagon are produced to form angle x. What is x?



Parallel lines



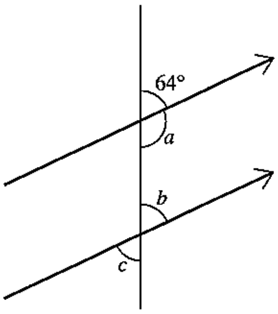
(i) $\hat{a} = \hat{c}$ (corresponding angles)
(ii) $\hat{c} = \hat{d}$ (alternate angles)
(iii) $\hat{b} + \hat{c} = 180°$ (allied angles)

Remember: 'The acute angles (angles less than 90°) are the same and the obtuse angles (angles between 90° and 180°) are the same.'

Exercise 3

In questions **1** to **9** find the angles marked with letters.

1.



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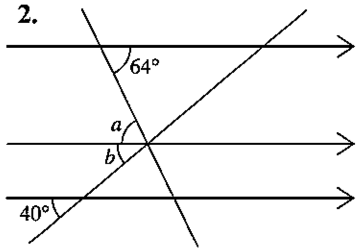

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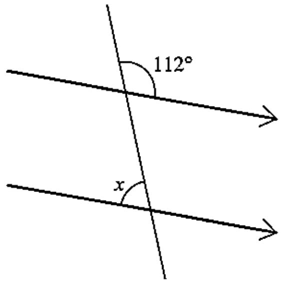

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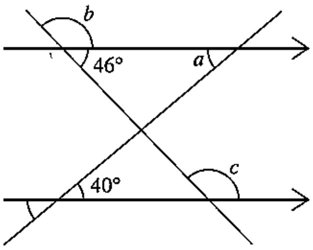

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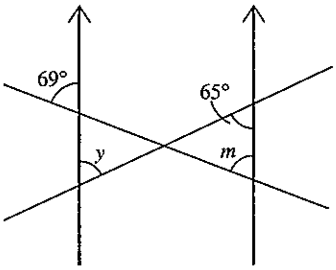

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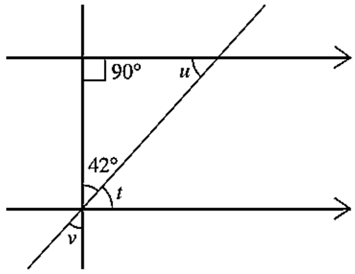

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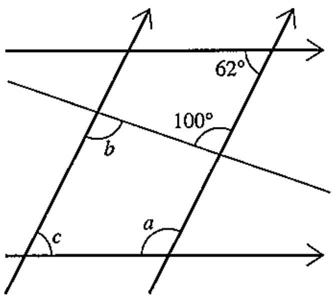

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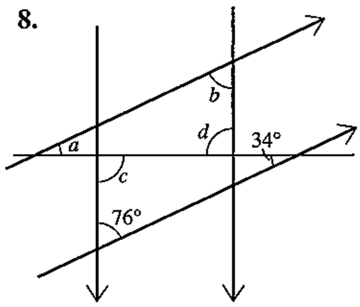

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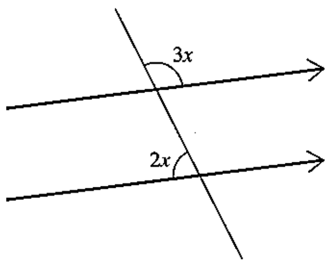
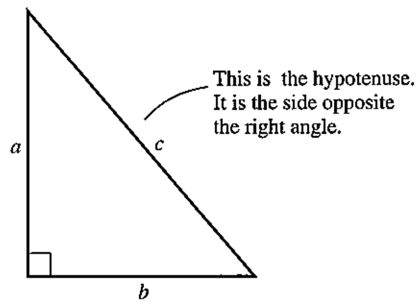


4.2 Pythagoras' theorem

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

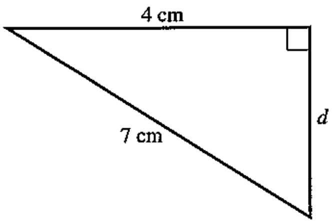
$a^2 + b^2 = c^2$



Example

Find the side marked d.

$$d^2 + 4^2 = 7^2$$
$$d^2 = 49 - 16$$
$$d = \sqrt{33} = 5{\cdot}74\,\text{cm (3 sig. fig.)}$$

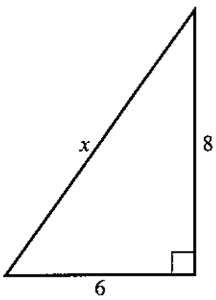


The *converse* is also true:
'If the square on one side of a triangle is equal to the sum of the squares on the other two sides, then the triangle is right-angled.'

Exercise 4

In questions **1** to **10**, find x. All the lengths are in cm.

1.



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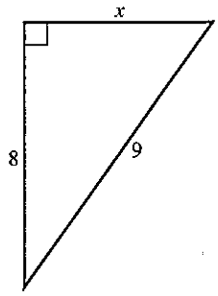

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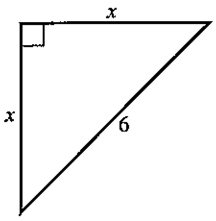

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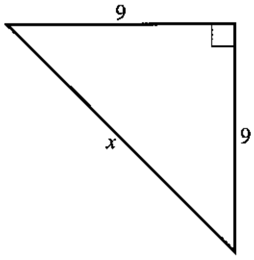

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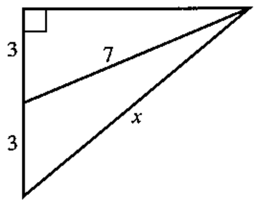

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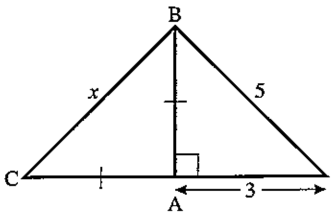

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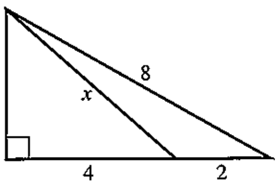

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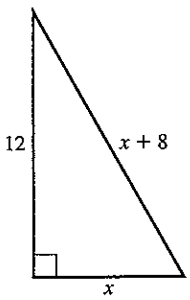

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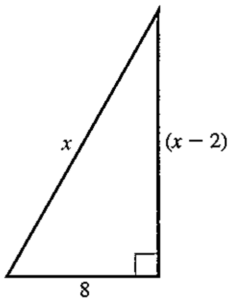

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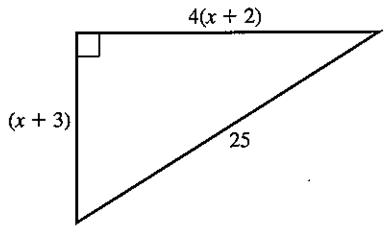


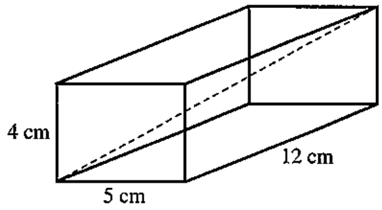
11. Find the length of a diagonal of a rectangle of length 9 cm and width 4 cm.

12. A square has diagonals of length 10 cm. Find the sides of the square.

13. A 4 m ladder rests against a vertical wall with its foot 2 m from the wall. How far up the wall does the ladder reach?

14. A ship sails 20 km due North and then 35 km due East. How far is it from its starting point?

15. Find the length of a diagonal of a rectangular box of length 12 cm, width 5 cm and height 4 cm.



16. Find the length of a diagonal of a rectangular room of length 5 m, width 3 m and height 2·5 m.

17. Find the height of a rectangular box of length 8 cm, width 6 cm where the length of a diagonal is 11 cm.

18. An aircraft flies equal distances South-East and then South-West to finish 120 km due South of its starting-point. How long is each part of its journey?

19. The diagonal of a rectangle exceeds the length by 2 cm. If the width of the rectangle is 10 cm, find the length.

20. A cone has base radius 5 cm and *slant* height 11 cm. Find its vertical height.

21. It is possible to find the sides of a right-angled triangle, with lengths which are whole numbers, by substituting different values of x into the expressions:

(a) $2x^2 + 2x + 1$ (b) $2x^2 + 2x$ (c) $2x + 1$

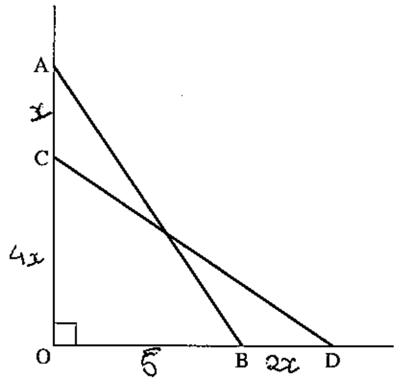
((a) represents the hypotenuse, (b) and (c) the other two sides.)

(i) Find the sides of the triangles when $x = 1, 2, 3, 4$ and 5.

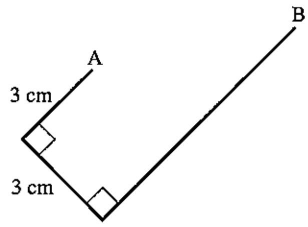
(ii) Confirm that $(2x+1)^2 + (2x^2+2x)^2 = (2x^2+2x+1)^2$

22. The diagram represents the starting position (AB) and the finishing position (CD) of a ladder as it slips. The ladder is leaning against a vertical wall.

Given: AC $= x$, OC $=$ 4AC, BD $=$ 2AC and OB $= 5$ m. Form an equation in x, find x and hence find the length of the ladder.



23. A thin wire of length 18 cm is bent into the shape shown. Calculate the length from A to B.

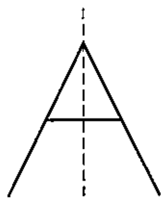


24. An aircraft is vertically above a point which is 10 km West and 15 km North of a control tower. If the aircraft is 4000 m above the ground, how far is it from the control tower?

4.3 Symmetry

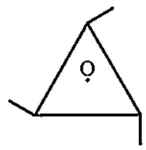
Line symmetry

The letter A has one line of symmetry, shown dotted.



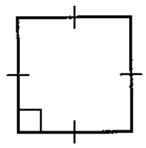
Rotational symmetry

The shape may be turned about O into three identical positions. It has rotational symmetry of order 3.

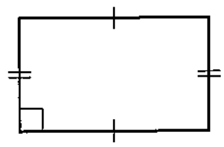


Quadrilaterals

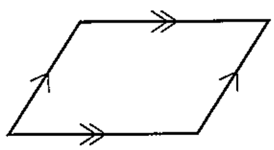
1. *Square*
   all sides are equal, all angles 90°, opposite sides parallel; diagonals bisect at right angles.



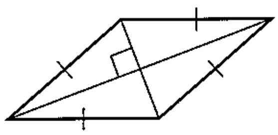
2. *Rectangle*
   opposite sides parallel and equal, all angles 90°, diagonals bisect each other.



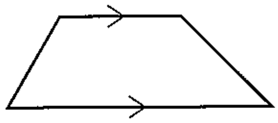
3. *Parallelogram*
   opposite sides parallel and equal, opposite angles equal, diagonals bisect each other (but not equal).



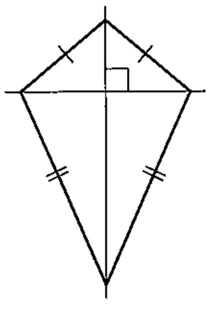
4. *Rhombus*
   a parallelogram with all sides equal, diagonals bisect each other at right angles and bisect angles.



5. *Trapezium*
   one pair of sides is parallel.

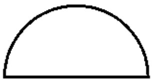
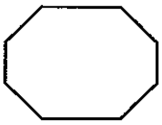
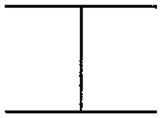
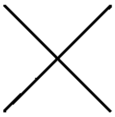
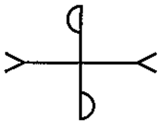
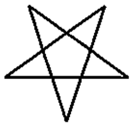
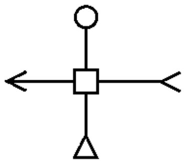
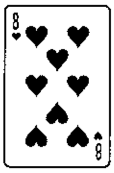
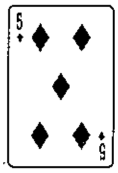
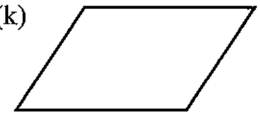
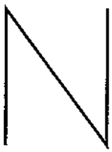
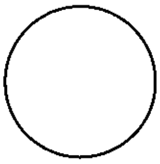
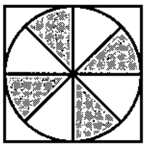


6. *Kite*
   two pairs of adjacent sides equal, diagonals meet at right angles bisecting one of them.

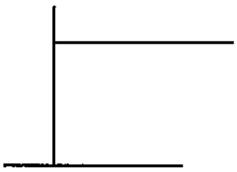


Exercise 5

1. For each shape state:
(a) the number of lines of symmetry (b) the order of rotational symmetry.

(a) 
(b) 
(c) 
(d) 
(e) 
(f) 
(g) 
(h) 
(i) 
(j) 
(k) 
(l) 
(m) 
(n) 

2. Add one line to each of the diagrams below so that the resulting figure has rotational symmetry but not line symmetry.

(a) 
(b) 

3. Draw a hexagon with just two lines of symmetry.

4. For each of the following shapes, find:
(a) the number of lines of symmetry
(b) the order of rotational symmetry.

square; rectangle; parallelogram; rhombus; trapezium; kite; equilateral triangle; regular hexagon.

In questions **5** to **15**, begin by drawing a diagram.

5. In a rectangle KLMN, $L\hat{N}M = 34°$. Calculate:
(a) $K\hat{L}N$ (b) $K\hat{M}L$

6. In a trapezium ABCD; $A\hat{B}D = 35°$, $B\hat{A}D = 110°$ and AB is parallel to DC. Calculate:
(a) $A\hat{D}B$ (b) $B\hat{D}C$

7. In a parallelogram WXYZ, $W\hat{X}Y = 72°$, $Z\hat{W}Y = 80°$. Calculate:
(a) $W\hat{Z}Y$ (b) $X\hat{W}Z$ (c) $W\hat{Y}Z$

8. In a kite ABCD, AB = AD; BC = CD; $C\hat{A}D = 40°$ and $C\hat{B}D = 60°$. Calculate:
(a) $B\hat{A}C$ (b) $B\hat{C}A$ (c) $A\hat{D}C$

9. In a rhombus ABCD, $A\hat{B}C = 64°$. Calculate:
(a) $B\hat{C}D$ (b) $A\hat{D}B$ (c) $B\hat{A}C$

10. In a rectangle WXYZ, M is the mid-point of WX and $Z\hat{M}Y = 70°$. Calculate:
(a) $M\hat{Z}Y$ (b) $Y\hat{M}X$

11. In a trapezium ABCD, AB is parallel to DC, AB = AD, BD = DC and $B\hat{A}D = 128°$. Find:
(a) $A\hat{B}D$ (b) $B\hat{D}C$ (c) $B\hat{C}D$

12. In a parallelogram KLMN, KL = KM and $K\hat{M}L = 64°$. Find:
(a) $M\hat{K}L$ (b) $K\hat{N}M$ (c) $L\hat{M}N$

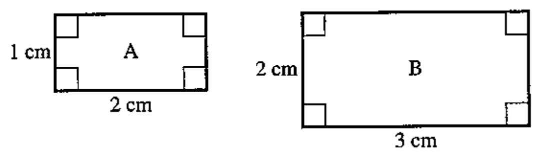
13. In a kite PQRS with PQ = PS and RQ = RS, $Q\hat{R}S = 40°$ and $Q\hat{P}S = 100°$. Find:
(a) $Q\hat{S}R$ (b) $P\hat{S}Q$ (c) $P\hat{Q}R$

14. In a rhombus PQRS, $R\hat{P}Q = 54°$. Find:
(a) $P\hat{R}Q$ (b) $P\hat{S}R$ (c) $R\hat{Q}S$

15. In a kite PQRS, $R\hat{P}S = 2P\hat{R}S$, PQ = QS = PS and QR = RS. Find:
(a) $Q\hat{P}S$ (b) $P\hat{R}S$ (c) $Q\hat{S}R$ (d) $P\hat{Q}R$

4.4 Similarity

Two triangles are similar if they have the same angles. For other shapes, not only must corresponding angles be equal, but also corresponding sides must be in the same proportion.
The two rectangles A and B are *not* similar even though they have the same angles.

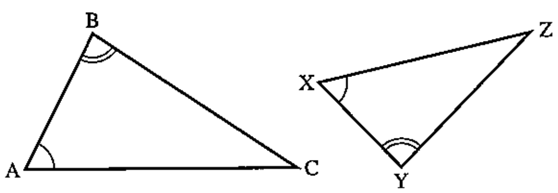


Example

In the triangles ABC and XYZ

$\hat{A} = \hat{X}$ and $\hat{B} = \hat{Y}$

so the triangles are similar. ($\hat{C}$ must be equal to $\hat{Z}$.)

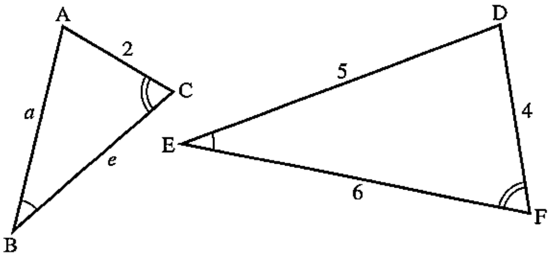


We have $\dfrac{BC}{YZ} = \dfrac{AC}{XZ} = \dfrac{AB}{XY}$

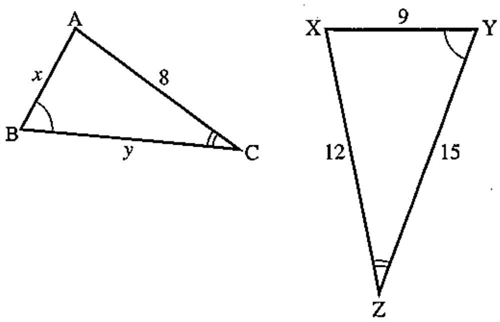
Exercise 6

Find the sides marked with letters in questions **1** to **11**; all lengths are given in centimetres.

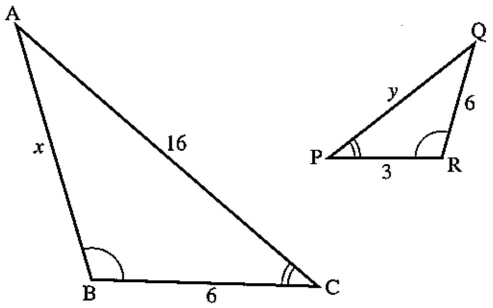
1.



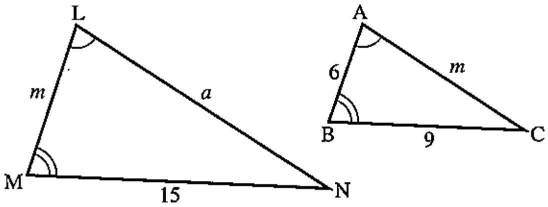
2.



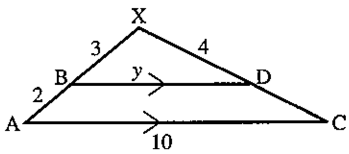
3.



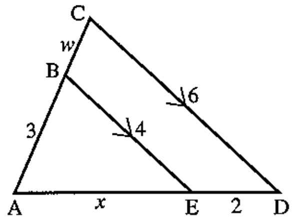
4.



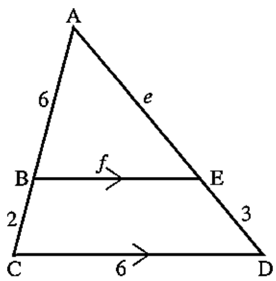
5.



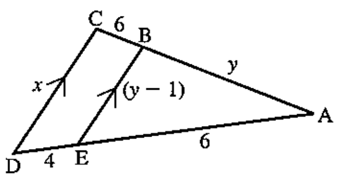
6.



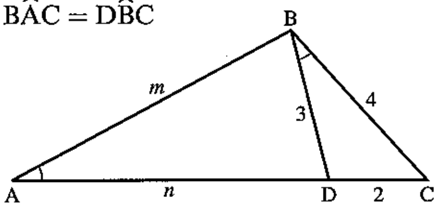
7.



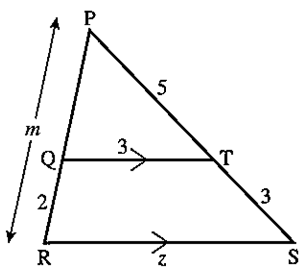
8.



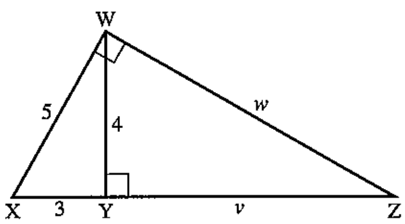
9. $B\widehat{A}C = D\widehat{B}C$



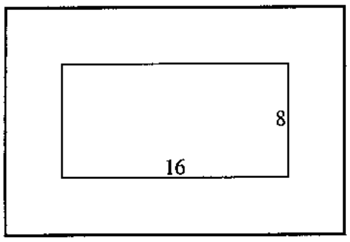
10.



11.



12. The drawing shows a rectangular picture 16 cm × 8 cm surrounded by a border of width 4 cm. Are the two rectangles similar?

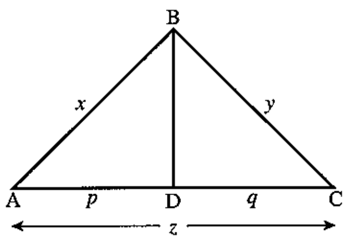


13. The diagonals of a trapezium ABCD intersect at O. AB is parallel to DC, AB = 3 cm and DC = 6 cm. If CO = 4 cm and OB = 3 cm, find AO and DO.

14. A tree of height 4 m casts a shadow of length 6·5 m. Find the height of a house casting a shadow 26 m long.

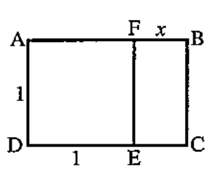
15. Which of the following *must* be similar to each other?
(a) two equilateral triangles
(b) two rectangles
(c) two isosceles triangles
(d) two squares
(e) two regular pentagons
(f) two kites
(g) two rhombuses
(h) two circles

16. In the diagram $A\hat{B}C = A\hat{D}B = 90°$, $AD = p$ and $DC = q$.
(a) Use similar triangles to show that $x^2 = pz$.
(b) Find a similar expression for y^2.
(c) Add the expressions for x^2 and y^2 and hence prove Pythagoras' theorem.



17. In a triangle ABC, a line is drawn parallel to BC to meet AB at D and AC at E. DC and BE meet at X. Prove that:
(a) the triangles ADE and ABC are similar
(b) the triangles DXE and BXC are similar
(c) $\dfrac{AD}{AB} = \dfrac{EX}{XB}$

18. From the rectangle ABCD a square is cut off to leave rectangle BCEF.
Rectangle BCEF is similar to ABCD. Find x and hence state the ratio of the sides of rectangle ABCD. ABCD is called the Golden Rectangle and is an important shape in architecture.

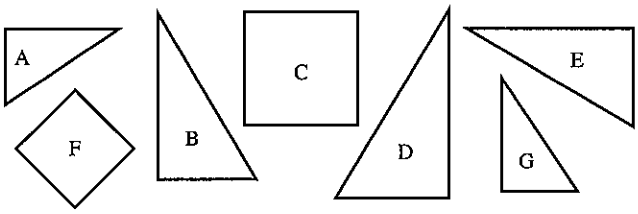


Congruence

Two plane figures are congruent if one fits exactly on the other. They must be the same size and the same shape.

Exercise 7

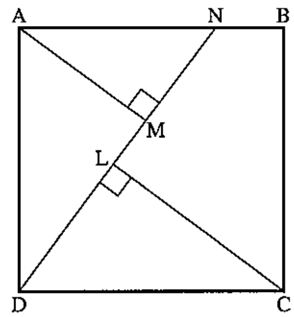
1. Identify pairs of congruent shapes below.



2. Triangle LMN is isosceles with LM = LN; X and Y are points on LM, LN respectively such that LX = LY. Prove that triangles LMY and LNX are congruent.

3. ABCD is a quadrilateral and a line through A parallel to BC meets DC at X. If $\hat{D} = \hat{C}$, prove that $\triangle ADX$ is isosceles.

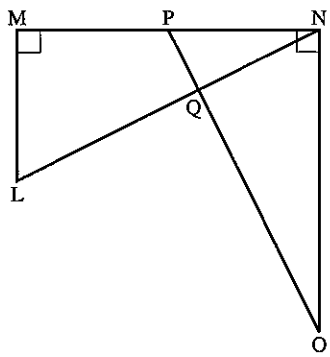
4. In the diagram, N lies on a side of the square ABCD, AM and LC are perpendicular to DN. Prove that:
   (a) $A\hat{D}N = L\hat{C}D$ (b) AM = LD



5. Points L and M on the side YZ of a triangle XYZ are drawn so that L is between Y and M. Given that XY = XZ and $Y\hat{X}L = M\hat{X}Z$, prove that YL = MZ.

6. Squares AMNB and AOPC are drawn on the sides of triangle ABC, so that they lie outside the triangle. Prove that MC = OB.

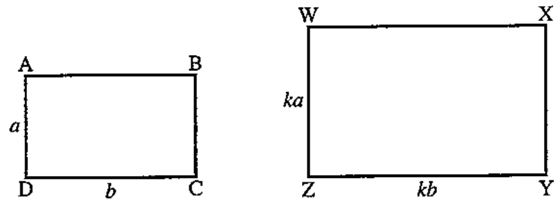
7. In the diagram, $L\hat{M}N = O\hat{N}M = 90°$. P is the mid-point of MN, MN = 2ML and MN = NO. Prove that:
   (a) the triangles MNL and NOP are congruent
   (b) $O\hat{P}N = L\hat{N}O$
   (c) $L\hat{Q}O = 90°$



8. PQRS is a parallelogram in which the bisectors of the angles P and Q meet at X. Prove that the angle PXQ is a right angle.

Areas of similar shapes

The two rectangles are similar, the ratio of corresponding sides being k.



area of ABCD $= ab$
area of WXYZ $= ka \times kb = k^2ab$

$$\therefore \quad \frac{\text{area WXYZ}}{\text{area ABCD}} = \frac{k^2ab}{ab} = k^2$$

This illustrates an important general rule for all similar shapes:

If two figures are similar and the ratio of corresponding sides is k, then the ratio of their areas is k^2.

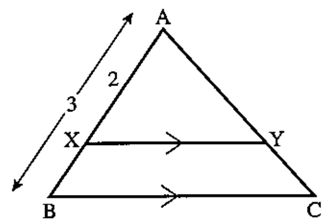
Note: k is sometimes called the *linear scale factor*.

This result also applies for the surface areas of similar three-dimensional objects.

Example 1

XY is parallel to BC.

$$\frac{AB}{AX} = \frac{3}{2}$$



If the area of $\triangle AXY = 4\,\text{cm}^2$, find the area of $\triangle ABC$.

The triangles ABC and AXY are similar.

Ratio of corresponding sides $(k) = \frac{3}{2}$

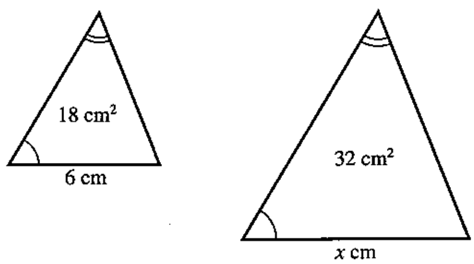
$\therefore$ Ratio of areas $(k^2) = \frac{9}{4}$

$\therefore$ Area of $\triangle ABC = \frac{9}{4} \times (\text{area of } \triangle AXY)$

$= \frac{9}{4} \times (4) = 9\,\text{cm}^2$

Example 2

Two similar triangles have areas of $18\,\text{cm}^2$ and $32\,\text{cm}^2$ respectively. If the base of the smaller triangle is 6 cm, find the base of the larger triangle.



Ratio of areas $(k^2) = \frac{32}{18} = \frac{16}{9}$

$\therefore$ Ratio of corresponding sides $(k) = \sqrt{\left(\frac{16}{9}\right)}$

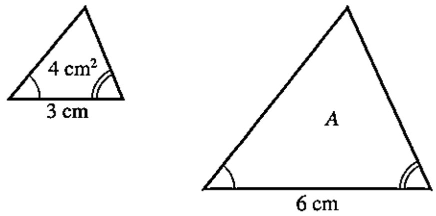
$= \frac{4}{3}$

$\therefore$ Base of larger triangle $= 6 \times \frac{4}{3} = 8\,\text{cm}$

Exercise 8

In this exercise a number written inside a figure represents the area of the shape in cm^2. Numbers on the outside give linear dimensions in cm. In questions **1** to **6** find the unknown area A. In each case the shapes are similar.

1.



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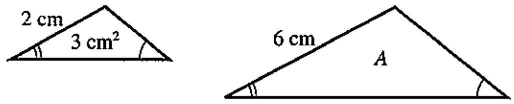

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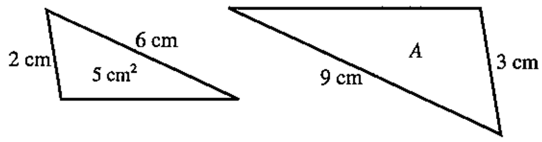

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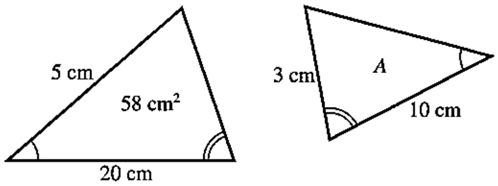

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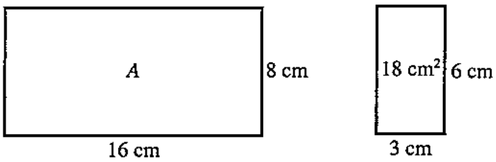

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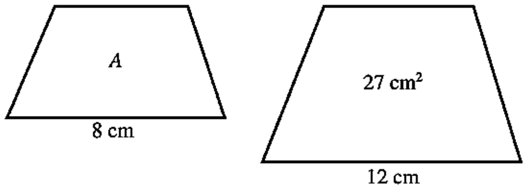
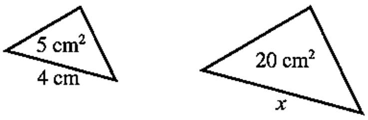


In questions **7** to **12**, find the lengths marked for each pair of similar shapes.

7.



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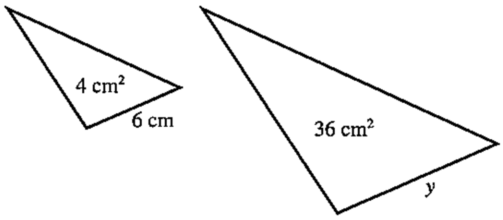

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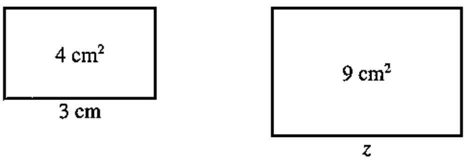

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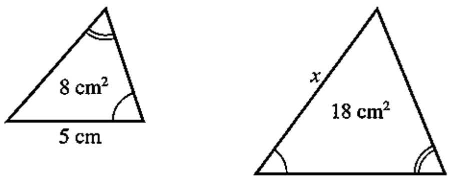

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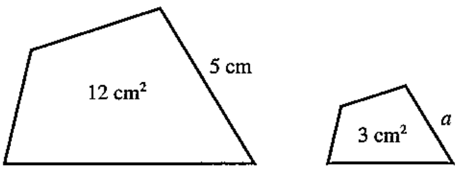

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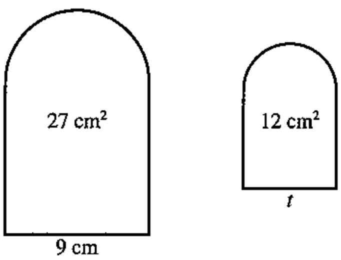
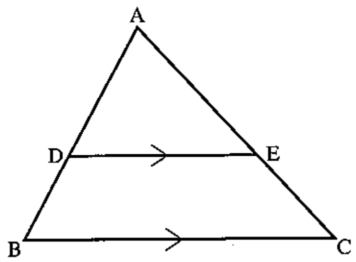
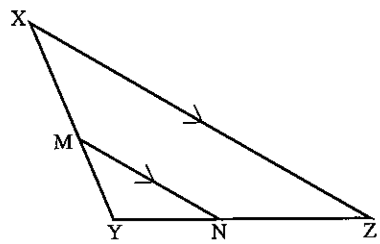


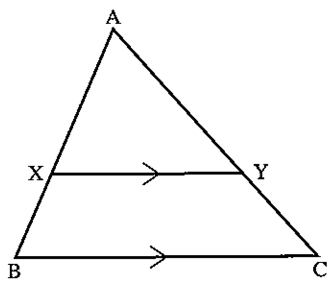
13. Given: AD = 3 cm, AB = 5 cm and area of $\triangle ADE = 6\,cm^2$.
Find:
(a) area of $\triangle ABC$ (b) area of DECB



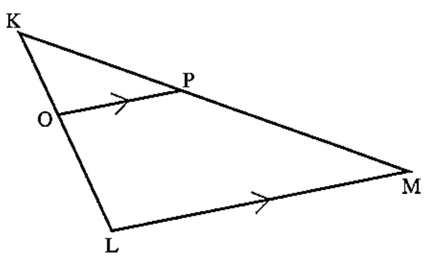
14. Given: XY = 5 cm, MY = 2 cm and area of $\triangle MYN = 4\,cm^2$.
Find:
(a) area of $\triangle XYZ$ (b) area of MNZX



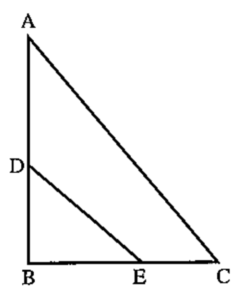
15. Given XY = 2 cm, BC = 3 cm and area of XYCB = $10\,cm^2$, find the area of $\triangle AXY$.



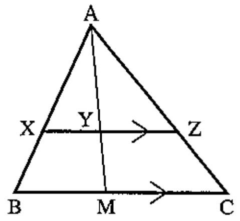
16. Given KP = 3 cm, area of $\triangle KOP = 2\,cm^2$ and area of OPML = $16\,cm^2$, find the length of PM.



17. The triangles ABC and EBD are similar (AC and DE are *not* parallel).
If AB = 8 cm, BE = 4 cm and the area of $\triangle DBE = 6\,cm^2$, find the area of $\triangle ABC$.



18. Given: AZ = 3 cm, ZC = 2 cm, MC = 5 cm, BM = 3 cm. Find:
    (a) XY
    (b) YZ
    (c) the ratio of areas AXY : AYZ
    (d) the ratio of areas AXY : ABM



19. A floor is covered by 600 tiles which are 10 cm by 10 cm. How many 20 cm by 20 cm tiles are needed to cover the same floor?
20. A wall is covered by 160 tiles which are 15 cm by 15 cm. How many 10 cm by 10 cm tiles are needed to cover the same wall?
21. When potatoes are peeled do you lose more peel or less when big potatoes are used as opposed to small ones?

Volumes of similar objects

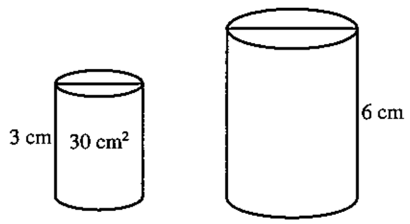
When solid objects are similar, one is an accurate enlargement of the other.
If two objects are similar and the ratio of corresponding sides is k, then the ratio of their volumes is k^3.

A line has one dimension, and the scale factor is used once.

An area has two dimensions, and the scale factor is used twice.

A volume has three dimensions, and the scale factor is used three times.

Example 1



Two similar cylinders have heights of 3 cm and 6 cm respectively. If the volume of the smaller cylinder is 30 cm^3, find the volume of the larger cylinder.

If linear scale factor = k, then ratio of heights $(k) = \frac{6}{3} = 2$

$$\therefore \quad \text{ratio of volumes } (k^3) = 2^3 = 8$$

$$\text{and volume of larger cylinder} = 8 \times 30 = 240\text{ cm}^3$$

Example 2

Two similar spheres made of the same material have weights of 32 kg and 108 kg respectively. If the radius of the larger sphere is 9 cm, find the radius of the smaller sphere.

We may take the ratio of weights to be the same as the ratio of volumes.

$$\text{ratio of volumes } (k^3) = \frac{32}{108} = \frac{8}{27}$$

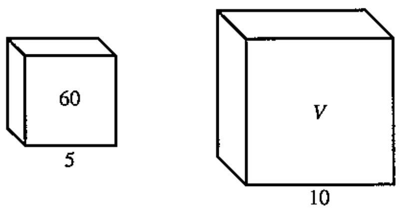
$$\text{ratio of corresponding lengths } (k) = \sqrt[3]{\left(\frac{8}{27}\right)} = \frac{2}{3}$$

$$\therefore \quad \text{Radius of smaller sphere} = \frac{2}{3} \times 9 = 6\,\text{cm}$$

Exercise 9

In this exercise, the objects are similar and a number written inside a figure represents the volume of the object in cm^3.
Numbers on the outside give linear dimensions in cm. In questions **1** to **8**, find the unknown volume V.

1.



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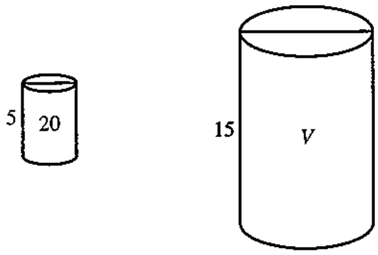

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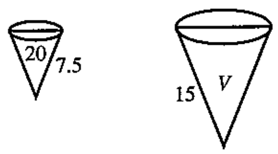

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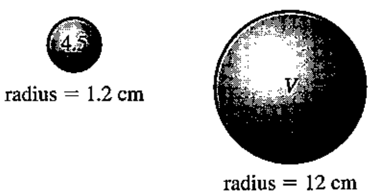

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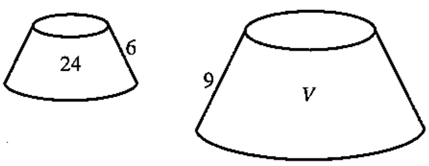

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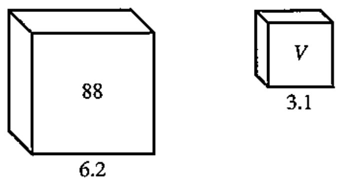

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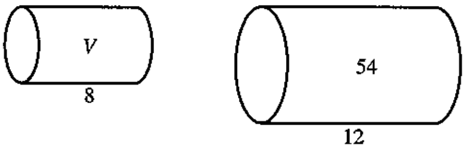

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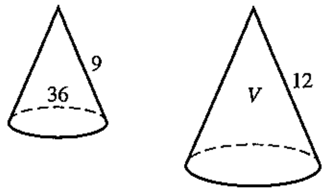
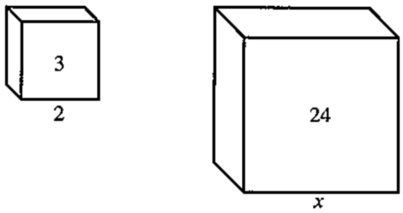


In questions **9** to **14**, find the lengths marked by a letter.

9.



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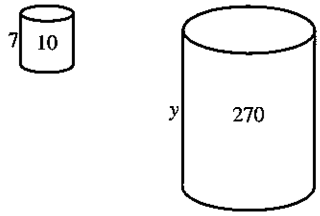

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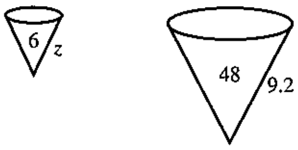

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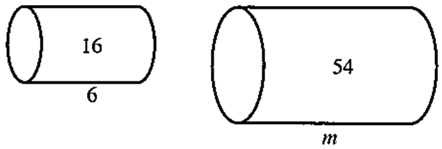

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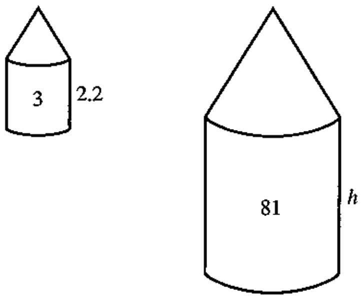


14.

15. Two similar jugs have heights of 4 cm and 6 cm respectively. If the capacity of the smaller jug is 50 cm^3, find the capacity of the larger jug.

16. Two similar cylindrical tins have base radii of 6 cm and 8 cm respectively. If the capacity of the larger tin is 252 cm^3, find the capacity of the small tin.

17. Two solid metal spheres have masses of 5 kg and 135 kg respectively. If the radius of the smaller one is 4 cm, find the radius of the larger one.

18. Two similar cones have surface areas in the ratio 4 : 9. Find the ratio of:
(a) their lengths, (b) their volumes.

19. The area of the bases of two similar glasses are in the ratio 4 : 25. Find the ratio of their volumes.

20. Two similar solids have volumes V_1 and V_2 and corresponding sides of length x_1 and x_2. State the ratio $V_1 : V_2$ in terms of x_1 and x_2.

21. Two solid spheres have surface areas of $5\,\text{cm}^2$ and $45\,\text{cm}^2$ respectively and the mass of the smaller sphere is 2 kg. Find the mass of the larger sphere.

22. The masses of two similar objects are 24 kg and 81 kg respectively. If the surface area of the larger object is $540\,\text{cm}^2$, find the surface area of the smaller object.

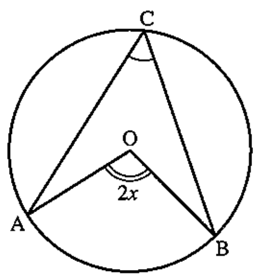
23. A cylindrical can has a circumference of 40 cm and a capacity of 4·8 litres. Find the capacity of a similar cylinder of circumference 50 cm.

24. A container has a surface area of $5000\,\text{cm}^2$ and a capacity of 12·8 litres. Find the surface area of a similar container which has a capacity of 5·4 litres.

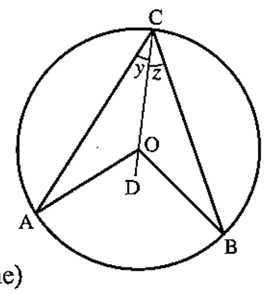
4.5 Circle theorems

(a) The angle subtended at the centre of a circle is twice the angle subtended at the circumference.

$$A\hat{O}B = 2 \times A\hat{C}B$$



Proof:
Draw the straight line COD.
Let $A\hat{C}O = y$ and $B\hat{C}O = z$.
In triangle AOC,



$AO = OC$ (radii)

$\therefore\ O\hat{C}A = O\hat{A}C$ (isosceles triangle)

$\therefore\ C\hat{O}A = 180 - 2y$ (angle sum of triangle)

$\therefore\ A\hat{O}D = 2y$ (angles on a straight line)

Similarly from triangle COB, we find

$D\hat{O}B = 2z$

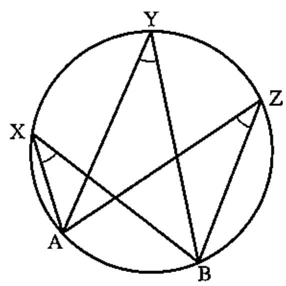
Now $A\hat{C}B = y + z$

and $A\hat{O}B = 2y + 2z$

$\therefore\ A\hat{O}B = 2 \times A\hat{C}B$ as required.

(b) Angles subtended by an arc in the same segment of a circle are equal.

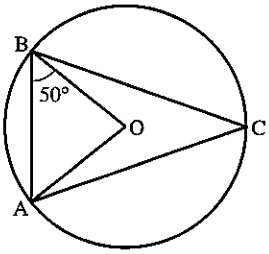
$$A\hat{X}B = A\hat{Y}B = A\hat{Z}B$$



Example 1

Given $A\hat{B}O = 50°$, find $B\hat{C}A$.
Triangle OBA is isosceles (OA = OB).

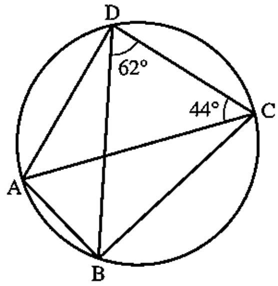
$\therefore$ $O\hat{A}B = 50°$
$\therefore$ $B\hat{O}A = 80°$ (angle sum of a triangle)
$\therefore$ $B\hat{C}A = 40°$ (angle at the circumference)



Example 2

Given $B\hat{D}C = 62°$ and $D\hat{C}A = 44°$, find $B\hat{A}C$ and $A\hat{B}D$.
$B\hat{D}C = B\hat{A}C$ (both subtended by arc BC)
$\therefore$ $B\hat{A}C = 62°$

$D\hat{C}A = A\hat{B}D$ (both subtended by arc DA)
$\therefore$ $A\hat{B}D = 44°$



Exercise 10

Find the angles marked with letters. A line passes through the centre only when point O is shown.

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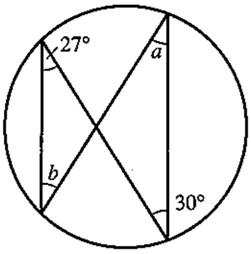

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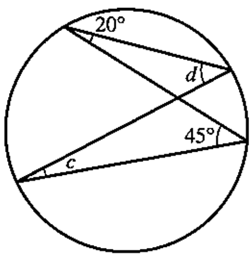

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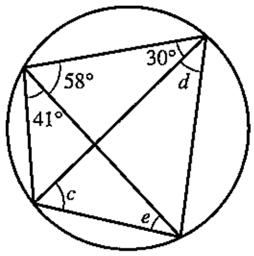

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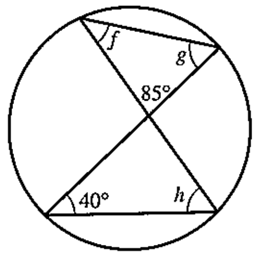

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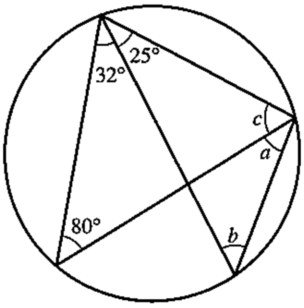

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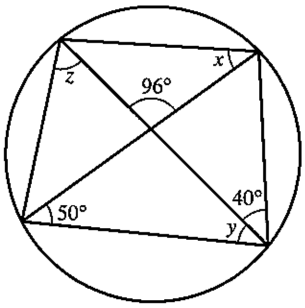

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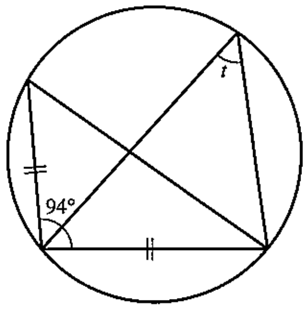

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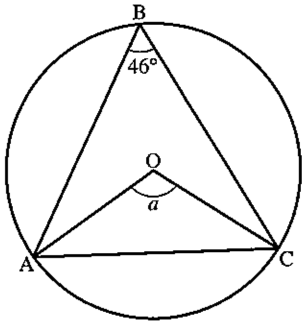

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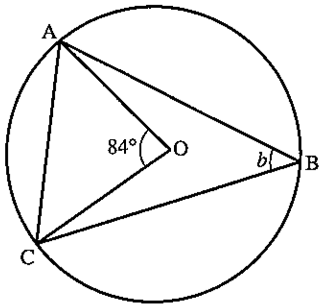

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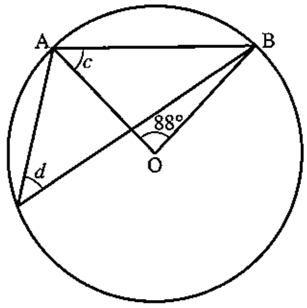

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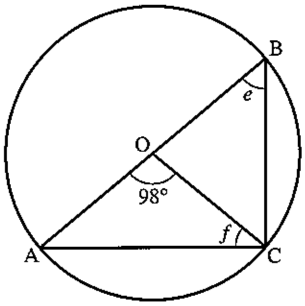

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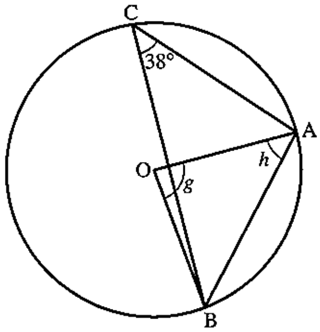

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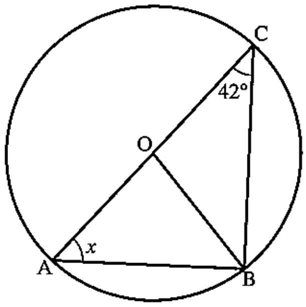

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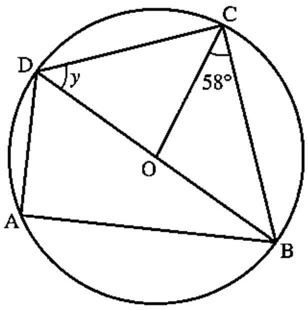

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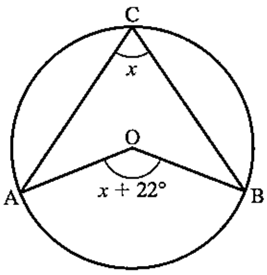

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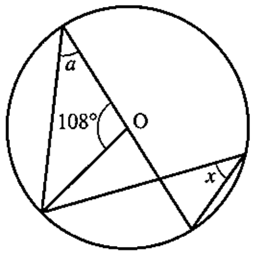
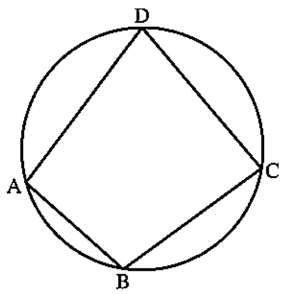


- ABCD is a cyclic quadrilateral. The corners touch the circle.

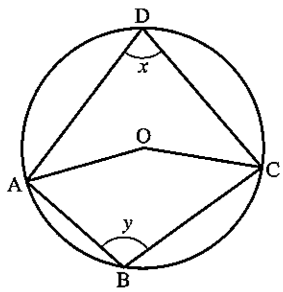
(c) The opposite angles in a cyclic quadrilateral add up to 180° (the angles are supplementary).

$$\hat{A} + \hat{C} = 180°$$
$$\hat{B} + \hat{D} = 180°$$



Proof:
Draw radii OA and OC.
Let $A\hat{D}C = x$ and $A\hat{B}C = y$.

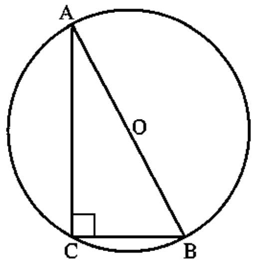
$A\hat{O}C$ obtuse $= 2x$ (angle at the centre)
$A\hat{O}C$ reflex $= 2y$ (angle at the centre)
$\therefore$ $2x + 2y = 360°$ (angles at a point)
$\therefore$ $x + y = 180°$ as required



(d) The angle in a semi-circle is a right angle.

In the diagram, AB is a diameter.

$A\hat{C}B = 90°$.



Example 1

Find a and x.

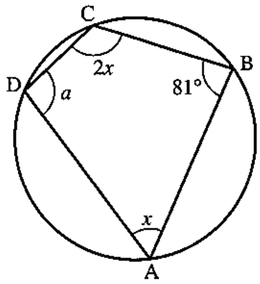
$a = 180° - 81°$ (opposite angles of a cyclic quadrilateral)

$\therefore \quad a = 99°$

$x + 2x = 180°$ (opposite angles of a cyclic quadrilateral)

$3x = 180°$

$x = 60°$



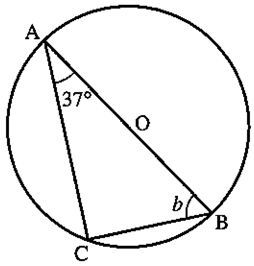
Example 2

Find b.

$A\hat{C}B = 90°$ (angle in a semi-circle)

$\therefore \quad b = 180° - (90 + 37)°$

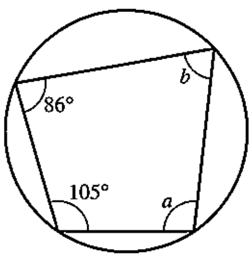
$= 53°$



Exercise 11

Find the angles marked with a letter.

1.



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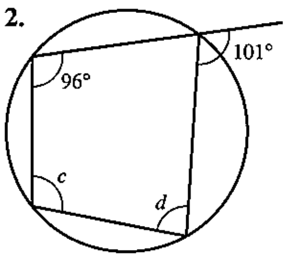

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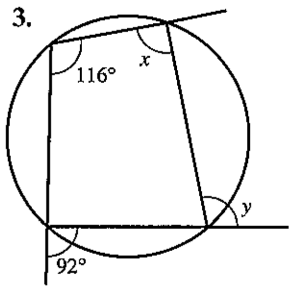

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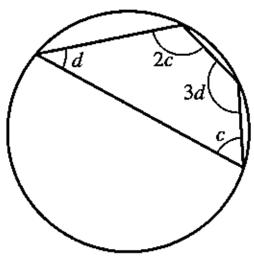

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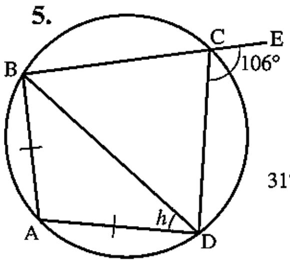

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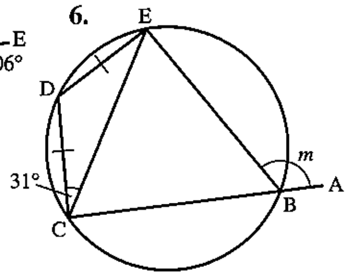

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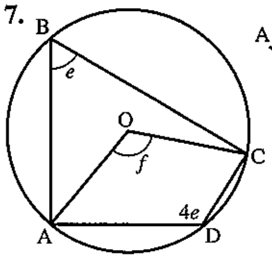

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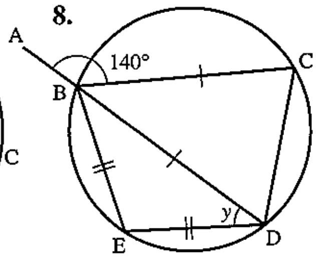

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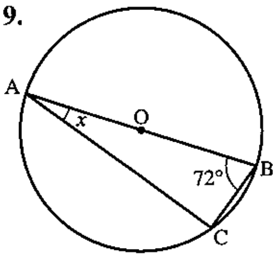

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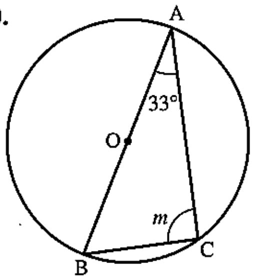

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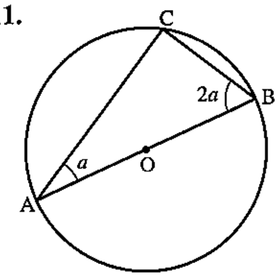

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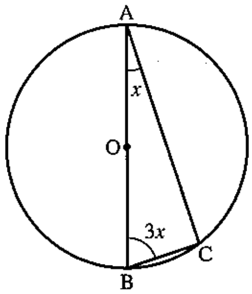

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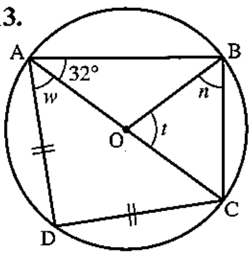

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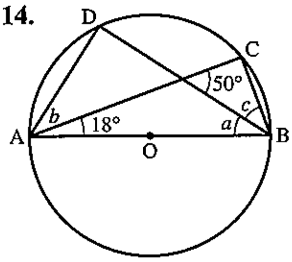

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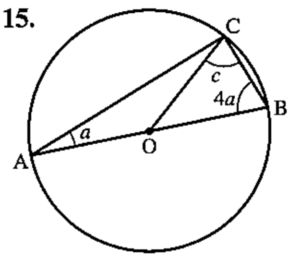

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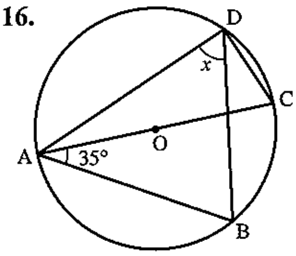

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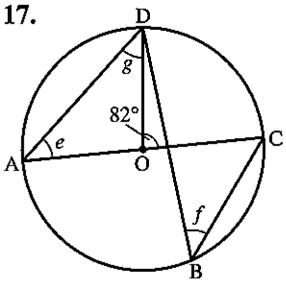

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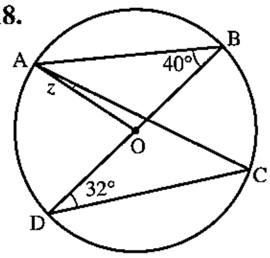

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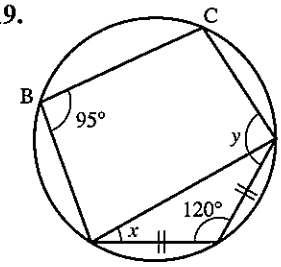

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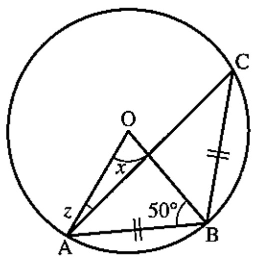
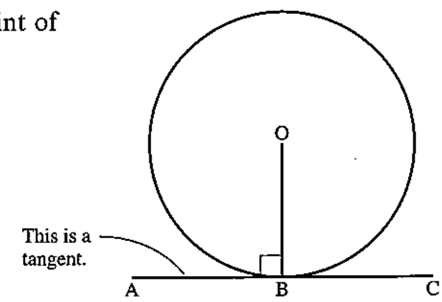


Tangents to circles

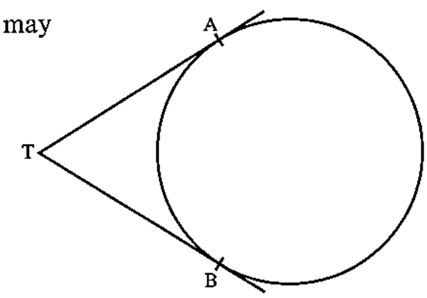
(a) The angle between a tangent and the radius drawn to the point of contact is 90°.

$A\widehat{B}O = 90°$



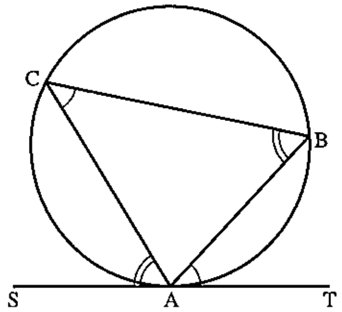
(b) From any point outside a circle just two tangents to the circle may be drawn and they are of equal length.

TA = TB



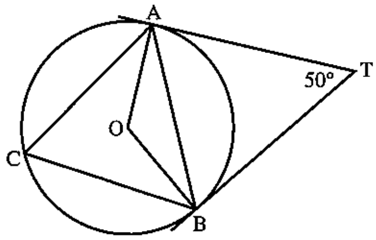
(c) Alternate segment theorem.
The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

$T\hat{A}B = B\hat{C}A$
and $S\hat{A}C = C\hat{B}A$



Example

TA and TB are tangents to the circle, centre O.
Given $A\hat{T}B = 50°$, find
(a) $A\hat{B}T$
(b) $O\hat{B}A$
(c) $A\hat{C}B$

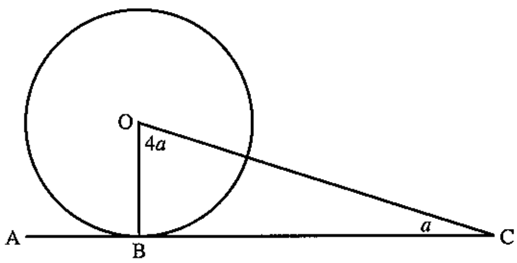


(a) $\triangle TBA$ is isosceles (TA = TB)
$\therefore \quad A\hat{B}T = \frac{1}{2}(180 - 50) = 65°$
(b) $O\hat{B}T = 90°$ (tangent and radius)
$\therefore \quad O\hat{B}A = 90 - 65$
$= 25°$
(c) $A\hat{C}B = A\hat{B}T$ (alternate segment theorem)
$A\hat{C}B = 65°$

Exercise 12

For questions **1** to **12**, find the angles marked with a letter.

1.



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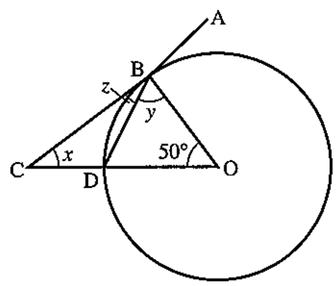

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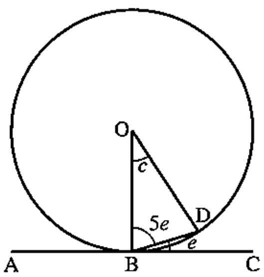

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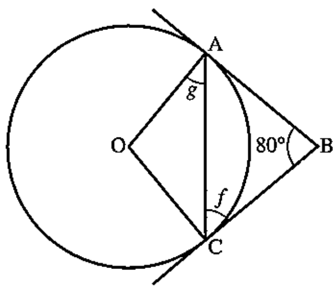

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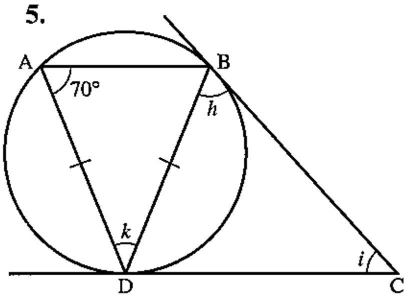

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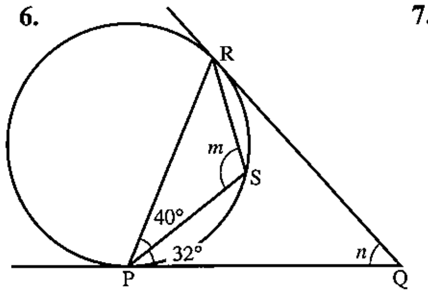

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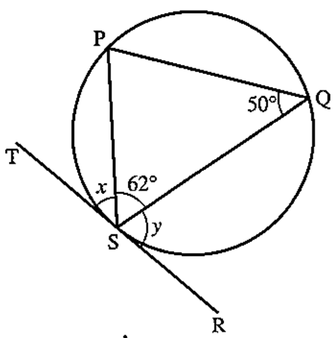

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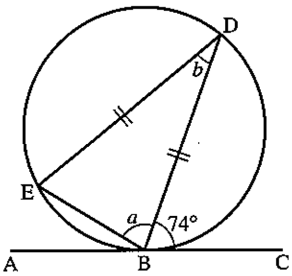

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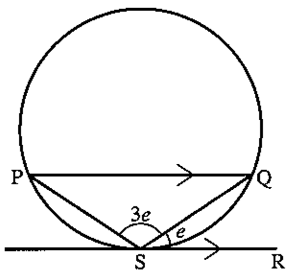

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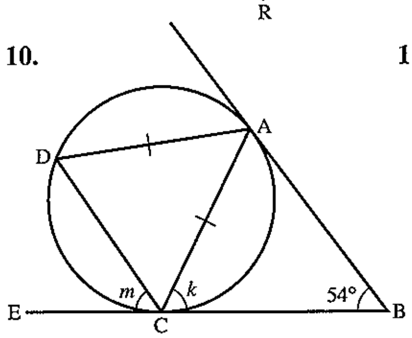

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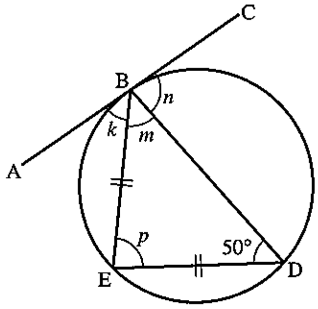

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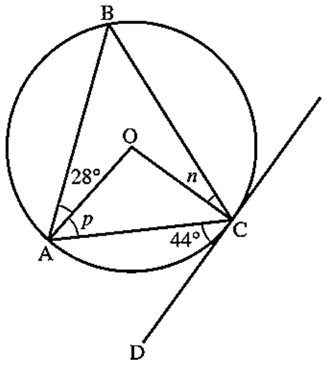
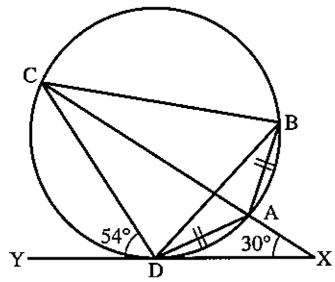
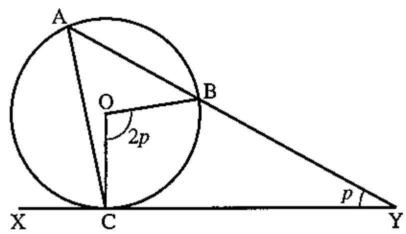


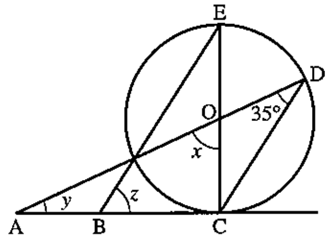
13. Find (a) $A\hat{D}X$ (b) $A\hat{B}C$ (c) $B\hat{C}D$



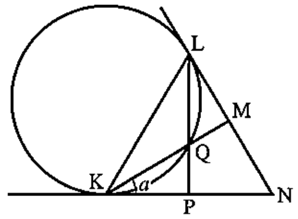
14. Find, in terms of p:
(a) $B\hat{A}C$ (b) $X\hat{C}A$ (c) $A\hat{C}O$



15. Find x, y and z.

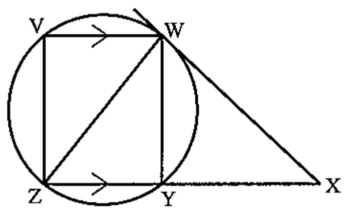


16. Given that KL = LN, LP bisects $K\hat{L}N$ and $M\hat{K}N = a$:



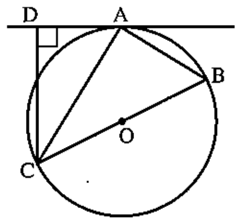
(a) prove that $\triangle$KLQ is isosceles
(b) find $L\hat{Q}M$ and $L\hat{M}Q$ in terms of a.

17. Show that:



(a) $Y\hat{W}X = V\hat{W}Z$
(b) the triangles VWZ and YWX are similar
(c) VW × WX = YW × WZ

18. Given that BOC is a diameter and that $A\hat{D}C = 90°$, prove that AC bisects $B\hat{C}D$.



19. The angles of a triangle are 50°, 60° and 70°, and a circle touches the sides at A, B, C. Calculate the angles of triangle ABC.

20. The tangents at A and B on a circle intersect at T, and C is any point on the major arc AB.

(a) If $A\hat{T}B = 52°$, calculate $A\hat{C}B$.
(b) If $A\hat{C}B = x$, find $A\hat{T}B$ in terms of x.

21. Line ATB touches a circle at T and TC is a diameter. AC and BC cut the circle at D and E respectively. Prove that the quadrilateral ADEB is cyclic.

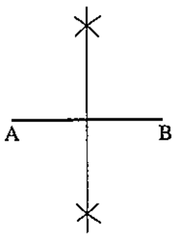
22. Two circles tough externally at T. A chord of the first circle XY is produced and touches the other at Z. The chord ZT of the second circle, when produced, cuts the first circle at W. Prove that $X\hat{T}W = Y\hat{T}Z$.

4.6 Constructions and loci

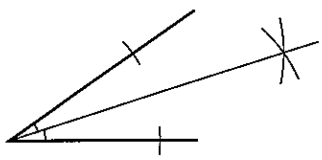
When the word 'construct' is used, the diagram should be drawn using equipment such as compasses, a ruler, a protractor etc.

Three basic constructions are shown below.

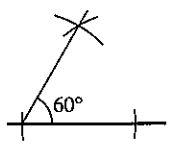
(a) Perpendicular bisector of a line joining two points



(b) Bisector of an angle



(c) 60° angle construction



Exercise 13

1. Construct a triangle ABC in which AB = 8 cm, AC = 6 cm and BC = 5 cm. Measure the angle $A\hat{C}B$.

2. Construct a triangle PQR in which PQ = 10 cm, PR = 7 cm and RQ = 6 cm. Measure the angle $R\hat{P}Q$.

3. Construct an equilateral triangle of side 7 cm.

4. Draw a line AB of length 10 cm. Construct the perpendicular bisector of AB.

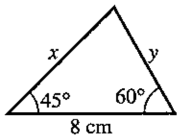
5. Draw two lines AB and AC of length 8 cm, where $B\hat{A}C$ is approximately 40°. Construct the line which bisects $B\hat{A}C$.

6. Draw a line AB of length 12 cm and draw a point X approximately 6 cm above the middle of the line. Construct the line through X which is perpendicular to AB.

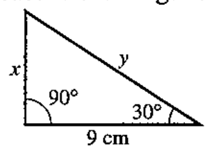
7. Construct an equilateral triangle ABC of side 9 cm. Construct a line through A to meet BC at 90° at the point D. Measure the length AD.

8. Construct the triangles shown and measure the length x.

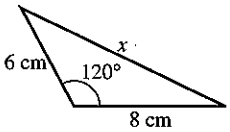
(a)



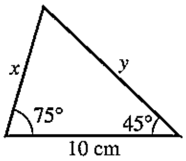
(b)



(c)



(d)



9. Construct a parallelogram WXYZ in which WX = 10 cm, WZ = 6 cm and $X\widehat{W}Z = 60°$. By construction, find the point A that lies on ZY and is equidistant from lines WZ and WX. Measure the length WA.

10. (a) Draw a line OX = 10 cm and construct an angle XOY = 60°.
(b) Bisect the angle XOY and mark a point A on the bisector so that OA = 7 cm.
(c) Construct a circle with centre A to touch OX and OY and measure the radius of the circle.

11. (a) Construct a triangle PQR with PQ = 8 cm, PR = 12 cm and $P\widehat{Q}R = 90°$.
(b) Construct the bisector of $Q\widehat{P}R$.
(c) Construct the perpendicular bisector of PR and mark the point X where this line meets the bisector of $Q\widehat{P}R$.
(d) Measure the length PX.

12. (a) Construct a triangle ABC in which AB = 8 cm, AC = 6 cm and BC = 9 cm.
(b) Construct the bisector of $B\widehat{A}C$.
(c) Construct the line through C perpendicular to CA and mark the point X where this line meets the bisector of $B\widehat{A}C$.
(d) Measure the lengths CX and AX.

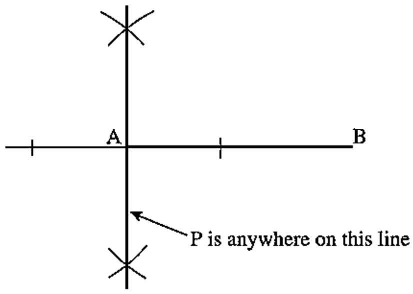
The locus of a point

The locus of a point is the path which it describes as it moves.

Example

Draw a line AB of length 8 cm.
Construct the locus of a point P which moves so that $B\widehat{A}P = 90°$.

Construct the perpendicular at A.
This line is the locus of P.

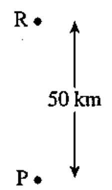


These are the basic loci you will come across:
1. Given distance from a given point. Locus is a circle.
2. Given distance from a straight line. Locus is a parallel line.
3. Equidistant from two given points. Locus is the perpendicular bisector of the line joining the two points.
4. Equidistant from two intersecting lines. Locus is the angle bisector of the two lines.

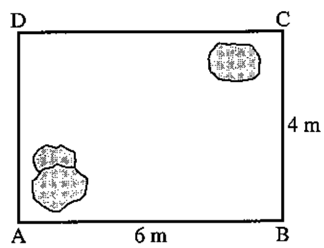
Exercise 14

1. Draw a line XY of length 10 cm. Construct the locus of a point which is equidistant from X and Y.
2. Draw two lines AB and AC of length 8 cm, where $B\hat{A}C$ is approximately 70°. Construct the locus of a point which is equidistant from the lines AB and AC.
3. Draw a circle, centre O, of radius 5 cm and draw a radius OA. Construct the locus of a point P which moves so that $O\hat{A}P = 90°$.
4. Draw a line AB of length 10 cm and construct the circle with diameter AB. Indicate the locus of a point P which moves so that $A\hat{P}B = 90°$.
5. (a) Describe in words the locus of M, the tip of the minute hand of a clock as the time changes from 3 o'clock to 4 o'clock.
   (b) Sketch the locus of H, the tip of the hour hand, as the time changes from 3 o'clock to 4 o'clock.
   (c) Describe the locus of the tip of the second hand as the time goes from 3 o'clock to 4 o'clock.
6. Inspector Clouseau has put a radio transmitter on a suspect's car, which is parked somewhere in Paris. From the strength of the signals received at points R and P, Clouseau knows that the car is
   (a) not more than 40 km from R, and
   (b) not more than 20 km from P.

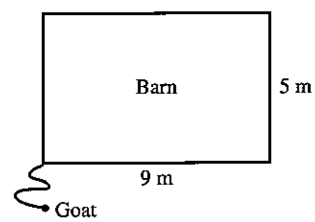
   Make a scale drawing [1 cm ≡ 10 km] and show the possible positions of the suspect's car.



7. A treasure is buried in the rectangular garden shown. The treasure is: (a) within 4 m of A and (b) more than 3 m from the line AD. Draw a plan of the garden and shade the points where the treasure could be.

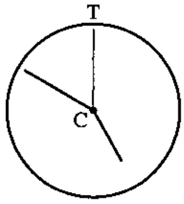


8. A goat is tied to one corner on the outside of a barn. The diagram shows a plan view. Sketch two plan views of the barn and show the locus of points where the goat can graze if
   (a) the rope is 4 m long,
   (b) the rope is 7 m long.



9. Draw a line AB of length 10 cm. With AB as base draw a triangle ABP so that the *area* of the triangle is 30 cm^2.
Describe the locus of P if P moves so that the area of the triangle ABP is always 30 cm^2.

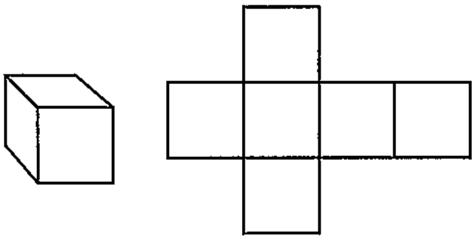
10. As the second hand of a clock goes through a vertical position, a money spider starts walking from C along the hand. After one minute the spider is at the top of the clock T. Describe the locus of the spider.



11. Sketch a side view of the locus of the valve on a bicycle wheel as the bicycle goes past in a straight line.

4.7 Nets

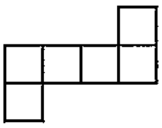
If the cube below was made of cardboard, and you cut along some of the edges and laid it out flat, you would have the *net* of the cube.



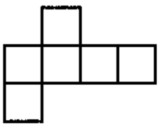
Exercise 15

1. Which of the nets below can be used to make a cube?

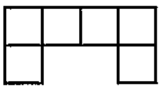
(a)



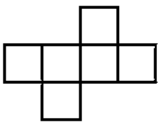
(b)



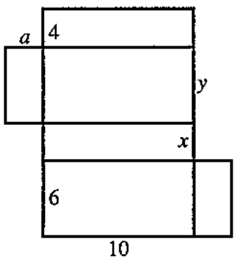
(c)



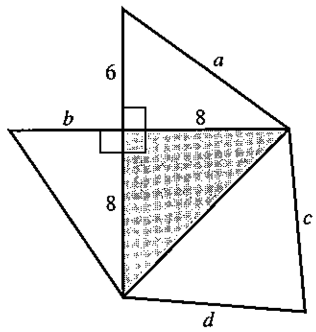
(d)



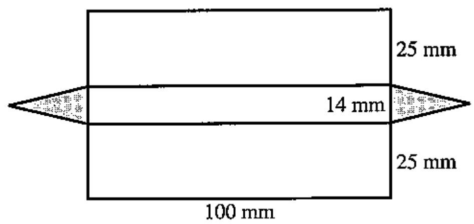
2. The diagram shows the net of a closed rectangular box. All lengths are in cm.
(a) Find the lengths a, x, y
(b) Calculate the volume of the box.



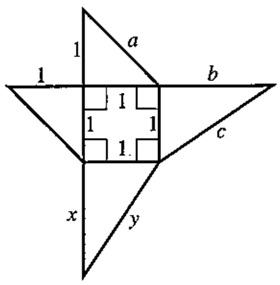
3. The diagram shows the net of a pyramid. The base is shaded. The lengths are in cm.
   (a) Find the lengths a, b, c, d
   (b) Find the volume of the pyramid.



4. The diagram shows the net of a prism.
   (a) Find the area of one of the triangular faces (shown shaded).
   (b) Find the volume of the prism.



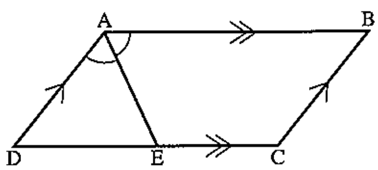
5. This is the net of a square-based pyramid.
   What are the lengths a, b, c, x, y?



6. Sketch nets for the following:
   (a) Closed rectangular box 7 cm × 9 cm × 5 cm.
   (b) Closed cylinder: length 10 cm, radius 6 cm.
   (c) Prism of length 12 cm, cross-section an equilateral triangle of side 4 cm.

Revision exercise 4A

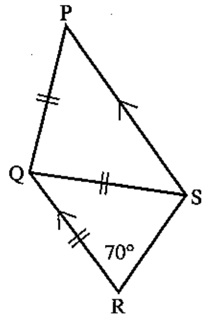
1. ABCD is a parallelogram and AE bisects angle A. Prove that DE = BC.



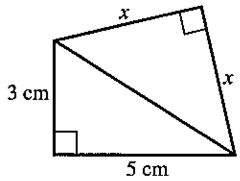
2. In a triangle PQR, $P\hat{Q}R = 50°$ and point X lies on PQ such QX = XR. Calculate $Q\hat{X}R$.

3. (a) ABCDEF is a regular hexagon. Calculate $F\hat{D}E$.
   (b) ABCDEFGH is a regular eight-sided polygon. Calculate $A\hat{G}H$.
   (c) Each interior angle of a regular polygon measures 150°. How many sides has the polygon?

4. In the quadrilateral PQRS, PQ = QS = QR, PS is parallel to QR and $Q\hat{R}S = 70°$. Calculate:
(a) $R\hat{Q}S$
(b) $P\hat{Q}S$



5. Find x.



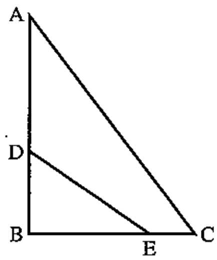
6. In the triangle ABC, AB = 7 cm, BC = 8 cm and $A\hat{B}C = 90°$. Point P lies inside the triangle such that BP = PC = 5 cm. Calculate:
(a) the perpendicular distance from P to BC
(b) the length AP

7. In triangle PQR the bisector of $P\hat{Q}R$ meets PR at S and the point T lies on PQ such that ST is parallel to RQ.
(a) Prove that QT = TS.
(b) Prove that the triangles PTS and PQR are similar.
(c) Given that PT = 5 cm and TQ = 2 cm, calculate the length of QR.

8. In the quadrilateral ABCD, AB is parallel to DC and $D\hat{A}B = D\hat{B}C$.
(a) Prove that the triangles ABD and DBC are similar.
(b) If AB = 4 cm and DC = 9 cm, calculate the length of BD.

9. A rectangle 11 cm by 6 cm is similar to a rectangle 2 cm by x cm. Find the two possible values of x.

10. In the diagram, triangles ABC and EBD are similar but DE is *not* parallel to AC. Given that AD = 5 cm, DB = 3 cm and BE = 4 cm, calculate the length of BC.

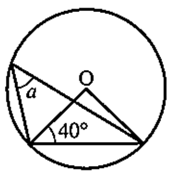


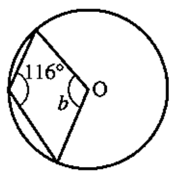
11. The radii of two spheres are in the ratio 2 : 5. The volume of the smaller sphere is $16\,cm^3$. Calculate the volume of the larger sphere.

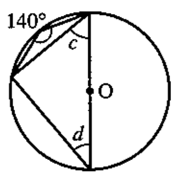
12. The surface areas of two similar jugs are $50\,cm^2$ and $450\,cm^2$ respectively.
(a) If the height of the larger jug is 10 cm, find the height of the smaller jug.
(b) If the volume of the smaller jug is $60\,cm^3$, find the volume of the larger jug.

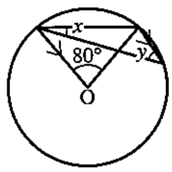
13. A car is an enlargement of a model, the scale factor being 10.
   (a) If the windscreen of the model has an area of $100\,\text{cm}^2$, find the area of the windscreen on the actual car (answer in m^2).
   (b) If the capacity of the boot of the car is $1\,\text{m}^3$, find the capacity of the boot on the model (answer in cm^3).

14. Find the angles marked with letters. (O is the centre of the circle.)

(a)


(b)


(c)


(d)


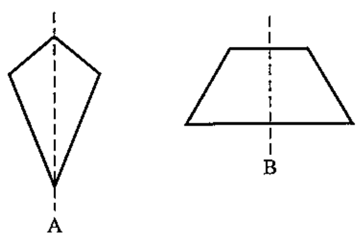
15. ABCD is a cyclic quadrilateral in which
AB = BC and $A\hat{B}C = 70°$. AD produced meets BC produced at the point P, where $A\hat{P}B = 30°$. Calculate:
(a) $A\hat{D}B$ (b) $A\hat{B}D$

16. Using ruler and compasses only:
   (a) Construct the triangle ABC in which AB = 7 cm, BC = 5 cm and AC = 6 cm.
   (b) Construct the circle which passes through A, B and C and measure the radius of this circle.

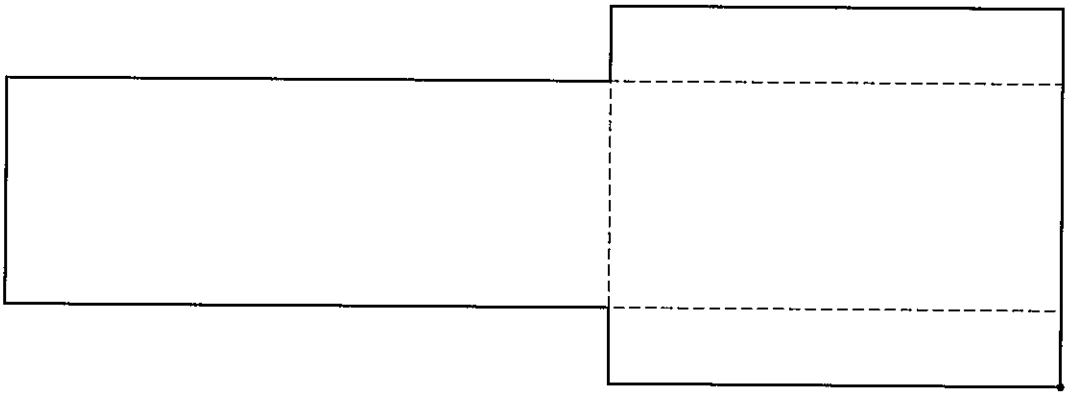
17. Construct:
   (a) the triangle XYZ in which XY = 10 cm, YZ = 11 cm and XZ = 9 cm.
   (b) the locus of points, inside the triangle, which are equidistant from the lines XZ and YZ.
   (c) the locus of points which are equidistant from Y and Z.
   (d) the circle which touches YZ at its mid-point and also touches XZ.

Examination exercise 4B

1. Two different quadrilaterals each have one, and only one, line of symmetry. In quadrilateral A, the line of symmetry is a diagonal. In quadrilateral B, the line of symmetry is not a diagonal.
Draw each of the quadrilaterals, showing the line of symmetry, and write down their special names. N 96 2



2.



(a) Copy the diagram and then draw two more broken lines to make it into the net of a cuboid.

(b) Mark clearly on the diagram the point A′ which will touch the point A when the net is folded to make the cuboid. J 97 2

3. Show, by drawing and shading, the set of all points which are at least 2 cm from a point O but no more than 3 cm from it. N 97 2

4. A 'Pythagorean triple' is a set of three whole numbers that could be the lengths of the three sides of a right-angled triangle.

(a) Show that $\{5, 12, 13\}$ is a Pythagorean triple.

(b) Two of the numbers in a Pythagorean triple are 24 and 25. Find the third number.

(c) The largest number in a Pythagorean triple is x and one of the other numbers is $x - 2$.

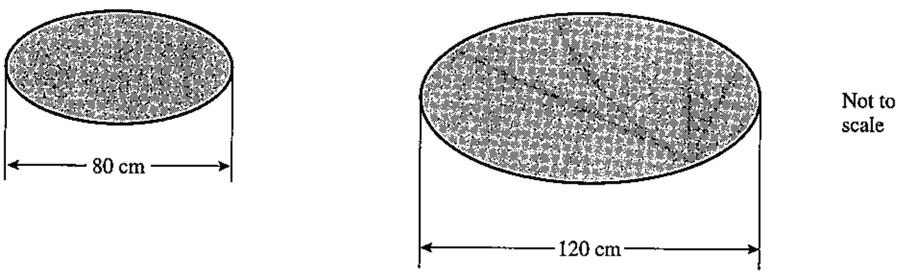
(i) If the third number is y, show that $y = \sqrt{4x - 4}$.

(ii) If $x = 50$, find the other two numbers in the triple.

(iii) If $x = 101$, find the other two numbers in the triple.

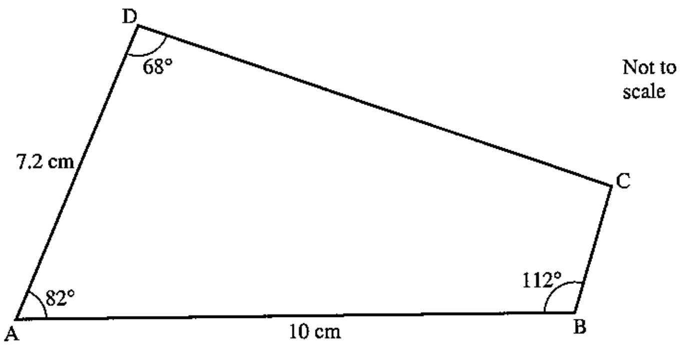
(iv) Find two other Pythagorean triples in the form $\{y, x - 2, x\}$, where $x < 40$. Remember that all three numbers must be whole numbers. N 97 4

5. Two table tops are similar in shape, as shown in the diagram.



The area of the smaller one is $\frac{1}{4}\,\text{m}^2$. Calculate the area of the larger one. N 97 2

6.



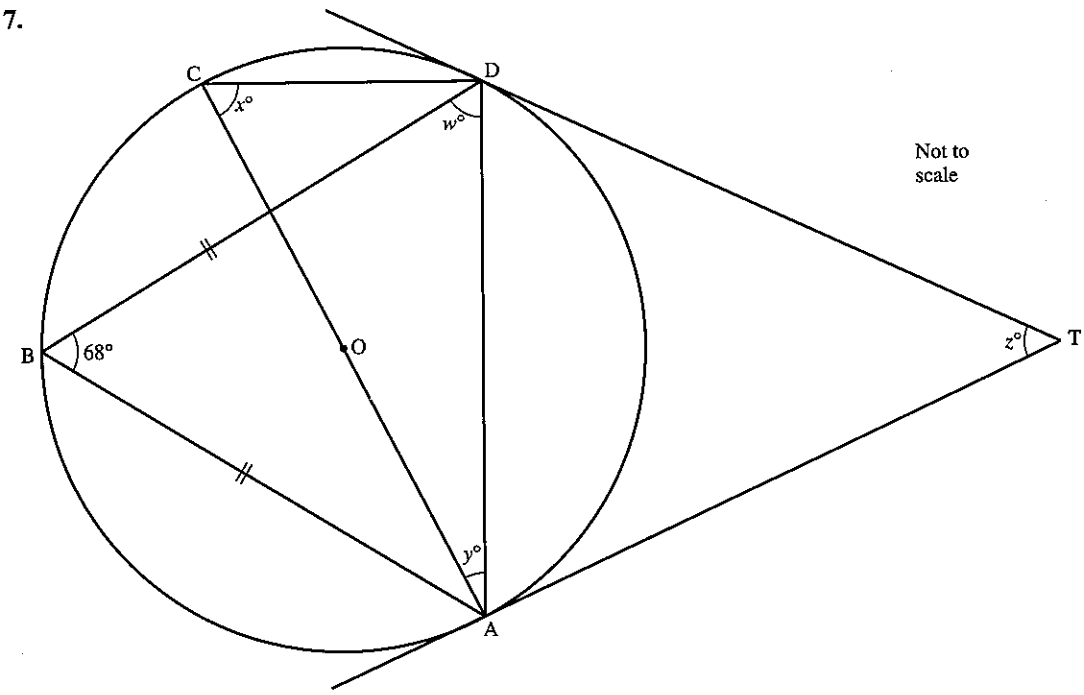
(a) Draw the line AB, 10 cm long, in the centre of a new page.
Construct a quadrilateral ABCD such that AD = 7·2 cm, angle DAB = 82°, angle ADC = 68° and angle ABC = 112°.

(b) Use a straight edge and compasses only to construct the perpendicular bisectors of AB and AD. These meet at E.
Leave all construction lines on your diagram.

(c) (i) Measure and write down the length AE.
(ii) Construct the locus of all points which are this distance from E.
(iii) Write down the special name of quadrilateral ABCD.

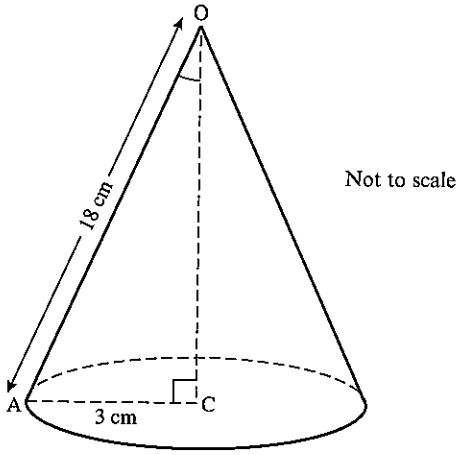
(d) Shade the region inside the quadrilateral which is nearer to A than it is to B, and nearer to A than it is to D. J 98 4

7.



AOC is the diameter of circle ABCD.
AT and DT are tangents. BD = BA and angle DBA = 68°.
Find the angles marked w, x, y and z. J 97 2

8. (a)



The diagram shows a hollow cone with base radius AC = 3 cm and edge OA = 18 cm. Calculate:

(i) the height OC,

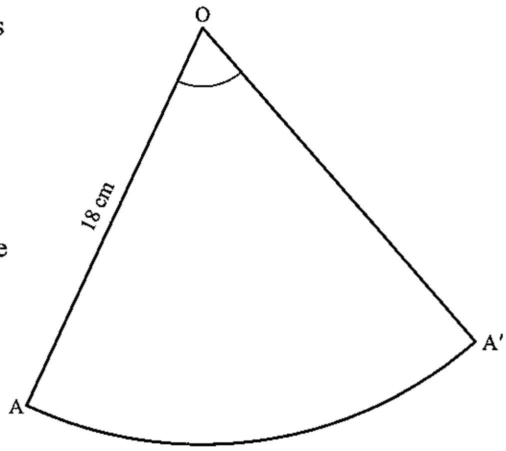
(ii) angle AOC,

(iii) the circumference of the base.
[π is approximately 3·142.]

(iv) The cone is cut along the line OA and opened out to form the sector AOA′. Calculate

(a) the circumference of a circle of radius 18 cm,

(b) angle AOA′.



(b) The top part of a solid cone is removed.
The height of the remaining solid is half the height of the original cone.

(i) Write down, in the form $1 : n$, the ratios:

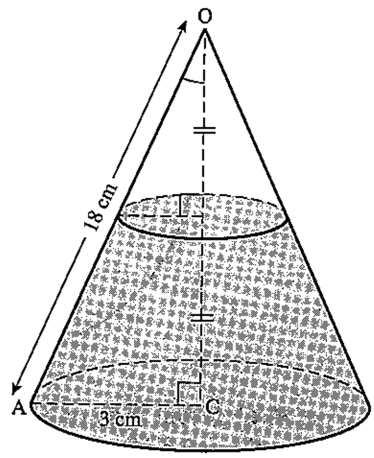
(a) the base radius of the cone removed : the base radius of the original cone,

(b) the curved surface area of the cone removed : the curved surface area of the original cone,

(c) the volume of the cone removed : the volume of the original cone.

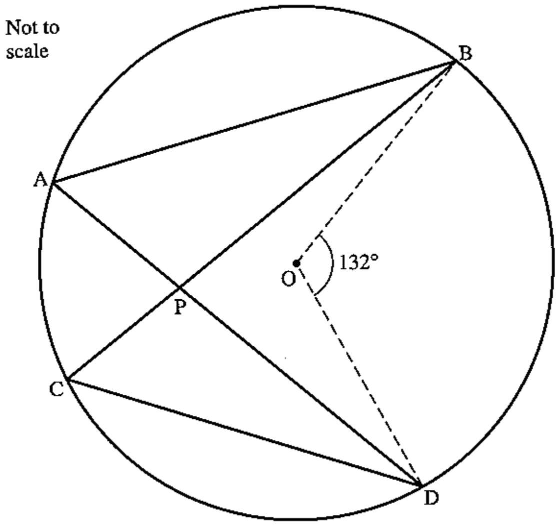
(ii) The curved surface area of the original cone was $24\pi\,\text{cm}^2$.
Calculate, in terms of π, the curved surface area of the remaining solid.

(iii) The volume of the original cone was $V\,\text{cm}^3$.
Give the volume of the remaining solid in terms of V.



J 95 4

9.



O is the centre of the circle. Angle BOD = 132°. The chords AD and BC meet at P.

(a) (i) Calculate angles BAD and BCD.

(ii) Explain why triangles ABP and CDP are similar.

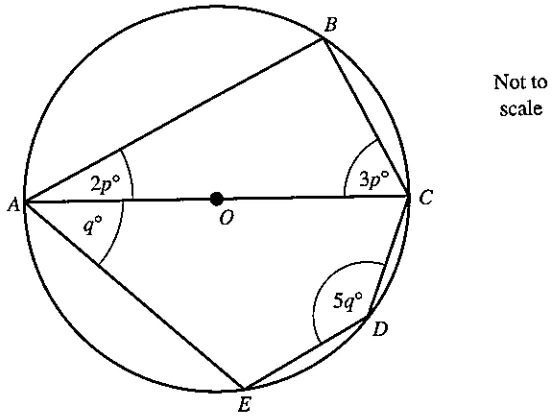
(iii) AP = 6 cm, PD = 8 cm, CP = 3 cm and AB = 17·5 cm. Calculate the lengths of PB and CD.

(iv) If the area of triangle ABP is $n\,\text{cm}^2$, write down, in terms of n, the area of triangle CPD.

(b) (i) The tangents at B and D meet at T. Calculate angle BTD.

(ii) Use OB = 9·5 cm to calculate the diameter of the circle which passes through O, B, T and D, giving your answer to the nearest centimetre. N 95 4

10.



A, B, C, D and E lie on a circle, centre O. AOC is a diameter.

Find the value of:

(a) p,

(b) q. J 03 2